Debt deflation, financial market, and regime change – Evidence from Europe*

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Abstract

The economic meltdown since 2008-9 has created disinflation, and even deflation in some countries in the Euro-area, in a period with large debt overhang, creating the condition for a continuing financial market crisis in the Euro-area. As disinflation and deflation push up the real interest rate, while growth and income declines, the leveraging problem becomes more severe and the economy risks shifting into a regime with high insolvency risk, high financial stress, rising credit spreads, possibly accompanied by strong adverse macroeconomic feedback loops. Investigating the consequences of those magnifying feedback loops, given the debt deflation, we demonstrate the possibility of unstable dynamics and downward spirals in the presence of regime-dependent macro feedback loops, using a theoretical model with decentralized matching mechanisms on both labor and financial markets. To explore the amplifying linkages between deflation, output, labor and financial markets, we employ a new solution procedure to solve our models variants for out-of-steady-state dynamics. We then empirically explore deflationary trends in Europe and employ a Global VAR (GVAR) model for a large euro area macro data set to estimate the impact of deflation on output. Moreover, we use a four variable Multi-Regime VAR (MRVAR) model with regime dependent IRs to study deflationary as well as well as the financial risk drivers in a MRVAR setting. New measures for financial risk drivers are employed and multi-regime IRs for output, inflation rates, interest rates and financial stress are explored. We also study regime changes in central macro relationships such as regime change in the credit - output link, the Phillips curve and in Okun's law.

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1 Introduction

The financial and economic meltdown and the large drop in output and employment after 2007-8 has created disinflation – and in some countries even a deflation – in a period with large debt overhang, invoking the Fisher debt deflation process of the 1930s. Yet, New Keynesian studies have found that neither the US nor the Euro-area has shown a collapse of the inflation rate despite the significant drop in output (growth). Much research started to explain why the inflation rates moved down so slowly and have not been falling as much as expected.[1] The process of disinflation and deflation in the Euro-area is slow.[2] Yet, does the EU face a period of debt deflation and the risk of a protracted recession? Moreover, will a regime of financial instability re-emerge, exacerbating deflationary risks?

There are a few recent studies exploring such scenarios. The study by Werning (2012), adapting a shorter horizon, deals with those issues in terms of a liquidity crisis. He employs a linear quadratic macro model with central banks minimizing the loss from output and inflation gap, and shows for the linearized macro model with an output equation and Phillips curve that as inflation turns into dis-inflation, or deflation, the real interest rate tends to rise and the liquidity trap emerges, even if the interest rate is already at the zero bound. This occurs with slowly moving prices, as in the New Keynesian model, and becomes worse with fully flexible prices. Yet in this rather short run model there is neither an evolution of debt nor a regime change in the financial market, possible amplifying the above process.

Another important recent study takes into account the evolution of debt and predicts a prolonged recession and a period of negative growth or slow growth in Europe, looking at debt build up and debt overhang (see Eggertsson and Krugman (EK), 2011[3]). EK in their work refer back to the Fisher debt-deflation study of the 1930s and more generally to the Fisher-Minksy-Koo approach. In EK, a sudden deleveraging shock will lead to falling prices, thus increasing real debt which in turn will decrease spending, thereby amplifying the adverse effects on prices – generating the typical downward spiral of a debt deflation. Yet the overall deflationary process seems to be slow.

Regarding the evolution in the US and the Euro area one might not go so far to invoke spiraling deflationary pressures, as the above two studies do, but what one obviously can observe is some debt overhang[4] and a dis-inflationary process in most countries, with some Southern European countries even sliding into a deflation. At the same time there is still severe, and sporadically rising, financial market stress, in particular in some countries in Southern Europe. The issue is then whether there will be a dis-inflationary process that triggers a debt deflation spiral, and to what extent will it be magnified through a jump to high financial stress, high credit spreads, as well as strong adverse macroeconomic feedback loops, which all could contribute to generate a protracted period of recession.[5]

Another important issue is that the Euro-area is characterized by significant heterogeneity. This suggests that US type monetary and fiscal policies will not capture sufficiently the diversity in the Euro-area. A uniform monetary and financial market policy as well as growth and labor market policies might be limited in their effects. For example QE for the entire Euro-area might overlook the specific bottlenecks in credit flows, quantity constraints, and default risk areas.[6]

In the spirit of the above studies, and the need for permitting more heterogeneity in the EU, we

[1] See Christiano et al. (2014).

[2] In the Great Depression the output level dropped from 1929 to 1932 by roughly 32 % and the price level declined in the same time period by 22%, see Marglin (2009), see also Fisher (1933). Though the drop in output in some Southern Euro-area countries was also high, prices dropped much less in the Euro-area since 2007-9, see figure 1.

[3] Some authors have discussed this also under the topic of a secular stagnation.

[4] A recent report of McKinsey (2015) seems to offer fresh evidence of this. Beside private and sovereign debt overhang, see Borio et al (2015), there is also significant bank debt overhang, see Schleer et al. (2014)

[5] There is related literature that maintains that a prolonged recession could be a result of a deep financial shocks with strongly affected banking system (Bordo et al. 2012) or as a result of a hysteresis effects after episodes of long term underutiliztion of capacity and unemployment. For further causes of low growth and prolonged recessions, due to slowing innovations, Gordon (2013), excess savings, Summers (2014) and multiple policy issues, Lo and Rogoff (2014). Yet as mostly agreed the leading cause seems to be the debt overhang, see Lo and Rogoff (2014), Jimeno (2014).

[6] See Brunnermeier and Sannikov (2014).

introduce a dynamic macro model which allows for decentralized matching mechanisms on labor and financial markets. This aspect gives the opportunity to consider the extensive heterogeneities in the EU.[7] Building on Ball and Mazumder (2014), and Gross and Semmler (2015), but similar to Werning (2012), we introduce a Phillips curve driving the rate of change of the inflation rate. Moreover, as in EK (2011), we allow the build-up of debt to be impacted by the price level. On the other hand, financial market stress can accelerate contractionary forces, and can prevent recoveries from taking place, leading to a prolonged recession and unemployment. We show that there may be both dis-inflationary – or deflationary – and credit market mechanisms working to produce such effects. Those forces are possibly creating macroeconomic instabilities and regime changes. Since the EK model assumes nominal debt contracts, deflation itself, the Fisher effect, will be a contractionary force. On the other hand, if debt contracts are in real terms, or represent inflation-adjusted one-period debt, the contractionary debt deflation effect might be reduced.[8] Yet, the financial market contraction may still be amplifying.

Those forces resulting from debt-overhang and disinflation/deflation are working through the product and asset price dynamics and then through credit channels. In our model, there is leveraged investment and borrowing by households from credit market, mediated through financial intermediaries, as well as bond issuing. Contracting credit markets and higher credit spreads – caused by previous excess leveraging and higher cost of borrowing – can create severe macro feedback effects and regime changes and financial market stress, so that households and firms also face credit constraints and rising credit spreads, such that overall aggregate demand tends to fall.[9]

When aggregate demand – and thus capital utilization rates – fall, the lower income generates lower net income to payoff liabilities, which in turn creates greater financial market stress, larger credit spreads, lower aggregate demand and so on. At the same time, on the price side, one can observe some disinflation or worse, deflation, affecting the above dynamics. Even though the nominal interest rate may be at a lower bound, if there is disinflation the real interest rates rises. If aggregate monetary policy cannot manage to accommodate a declining or negative real interest rate by increasing inflation, then a lower bound – possibly the zero-lower bound – of the interest rate binds, output stays low and unemployment rises.

The real and nominal forces that accelerate downturns possibly create lock-ins into a prolonged period of a recession. This is basically working as a positive feedback loop between the product market, price dynamics, credit and financial market and economic activity,[10] where there might be excess savings accompanied by a long lasting recession and unemployment, or even a secular stagnation, as Summers (2014) and others have predicted. In this sense it is not a deleveraging shock but rather a slow process of debt-disinflation or debt-deflation, accompanied by rising financial market stress and credit spreads, causing the long-lasting recession. This way monetary and fiscal policies face great challenges and might not be effective on the aggregate level.

Our matching mechanisms in the labor market follows Merz (1995) and Ernst and Semmler (2010). The matching mechanism on the credit market is handled through financial intermediation mechanisms, and similar to Wasmer and Weil (2004) and Cui et al. (2014). Both permit us to potentially allow for heterogeneity and to study the financial macro linkages in a multi-period model. But we do this without building on an infinite horizon model of the macroeconomy, where agents usually have rational expectations and smooth consumption in the infinite horizon context, and experience preference, technology and policy shocks, yet regime changes do not occur. Usually in this context, models are linearized and only small deviations from the steady state are allowed for, but large shocks are not accounted for.[11]

In contrast, our approach permits to study the credit-macro feedback mechanisms in a multi-

[7] In this study this aspect is only preliminarily explored, further work along those lines needs to be done.

[8] This has been an argument against the EK model.

[9] See Blanchard et al. (2013). Those positive feedback loops are already mentioned in Fisher (1933).

[10] This naturally shows up in some measures of capacity utilization. Many recent DSGE models have started working with endogenous capital utilization and financial market, for example cost of capital when issuing bonds; see for example Sugo and Ueda (2016). A relationship between capital utilization and the "user cost of capital" is also postulated by Keynes (1936).

[11] The models by Werning (2012) and also Eggertsson and Krugman (2012) are also infinite horizon models and solved through local linearizations.

period model without assuming an infinite time decision horizon. In this context we can then explore the impact of policy effects in typical regimes, such as booms and recessions. We then take the model to the data and estimate and apply a multi-regime VAR (MRVAR), as used in Ernst, Mittnik and Semmler (2011) and Mittnik and Semmler (2013), and Schleer and Semmler (2013, and Schleer et al. (2014).[12] But since we want to study also deflationary risk drivers, in addition to financial risk drivers, our MRVAR employed here is, however, higher dimensional and the MRVAR and the IRs work with four important macro variables such as output, inflation rate, credit spread and financial stress.

As to the solution method, our model will not be solved locally through local linearization about the steady state, as used in DYNARE, or globally by Dynamic Programming, as in Ernst and Semmler (2010), but by non-linear model predictive control (NMPC), which has recently been developed by Gruene and Pannek (2011) and applied in Gruene et al. (2015). This numerical method allows for approximating the accurate dynamic of the model by an N-period receding horizon model which will provide us with an approximate solution for the decision and state variables as well as for the value function. Though the NMPC numerical method approximates the infinite time horizon model, with time periods N becoming very large, the NMPC permits one to explore important issues, such as the rise of important constraints and regime switching, in a model with a shorter time horizon.

The remainder of the paper is as follows. The next section presents some stylized facts concerning debt-deflation dynamics and considers the differences between different world regions. Section 3 presents the theoretical model with decentralized labor and credit market matching mechanisms, that introduces inflation and the dynamics of the price level and their impact on the capital stock, leveraging, output gap and employment. Section 4 studies the model with endogenous regime change in the finance-macro link. In section 5 the higher dimensional MRVAR methodology is applied to detect nonlinearities and regime changes in the link between output, inflation rate, credit spread and financial stress. The IRs for shocks on inflation rates, interest rates, output, and financial stress are explored in an econometric regime-change model. Some policy conclusions are drawn in sect. 6 and sect. 7 concludes. Technical details and some explorations of regime dependent macro laws can be found in the appendix.

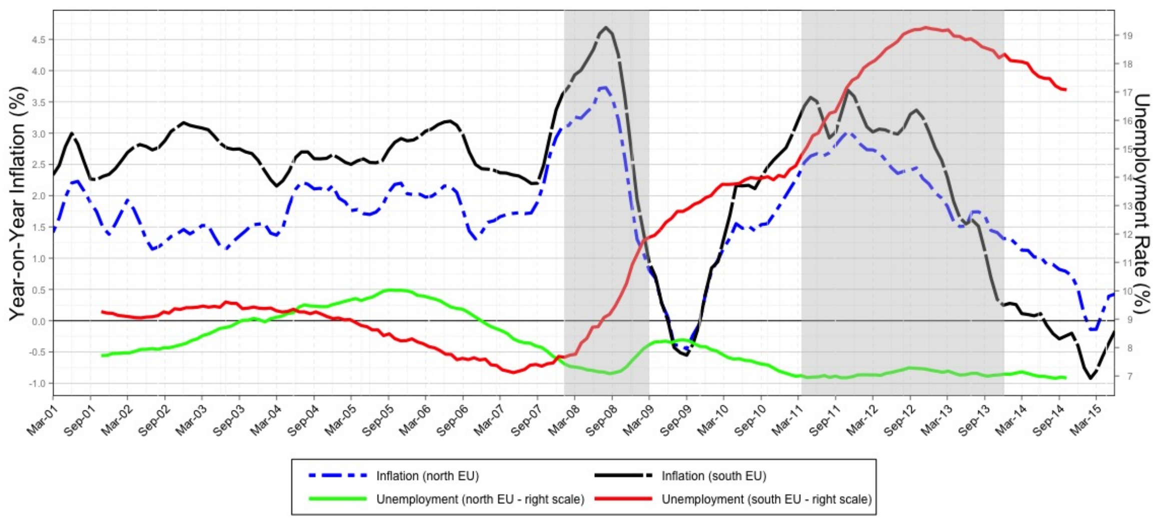
2 Stylized facts and GVAR results

Next we want to provide some stylized facts of the Euro-area and empirically explore to what extent there are precarious deflationary trends in the EU as compared to other regions of the world, for example the US and Japan. The latter can be undertaken by using a large scale Global Vector Autoregressive, called GVAR developed by Pesaran et al (2014) and applied in Binder and Gross (2014).

Let us first establish to what extent their has been disinflation or even deflationary pressures in the EU. Figure 1 demonstrates that there are particularly in the Southern countries (Italy, Spain, Portugal and Greece) deflationary pressures. The Northern countries (Germany, France, Austria) only show disinflation. Figure 1 plots also the unemployment rate for EU South and EU North. As can be seen, though the EU South region shows much sharper movements in the unemployment rate, the dis- or de-flation rate remains relatively modest– an issue we will explore below.

[12] We can allow for regime switching, as can be found in recent DSGE models, see Eo (2009), Schorfheide (2005) and Farmer et al (2008). There, however, it is assumed that the Euler equation, based on an infinite horizon solution, holds.

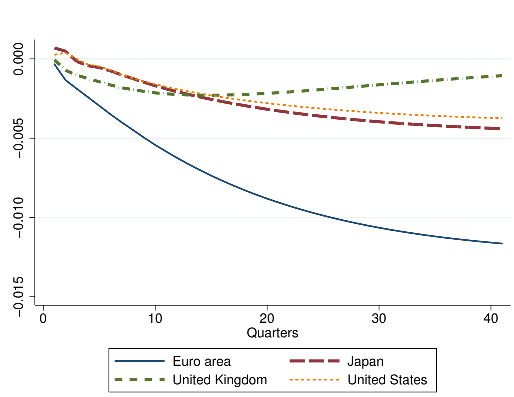
Figure 1: Comparison



Next we want to explore the relative importance of the deflationary mechanism for the EU, Japan, US and United Kingdom, using the GVAR methodology as put forward by Binder and Gross (2014). GVAR allows for a large scale econometric approach to model the economic interdependence in macro variables not only for time series data but also permits to model the interdependencies across countries. The interlinkages between countries can be studied by combining, via trade-weights, a set of country-specific VARs that contain weighted foreign variable vectors. This approach permits to model simultaneously a large number of countries, and a broad set of economic time series variables in one model. Usually if one models an unrestricted conventional VAR this is quite unfeasible due to the large number of parameters. The GVAR shows how one can set up and use multiple cross-sections, while, at the same time, one studies time series data of countries and regions. Variables employed here are inflation rates, GDP, equity market performance, short and long term interest rates, raw material and oil prices and exchange rates.[13]

The results of the GVAR study show that the EU has recently revealed stronger deflationary trends than the US, UK, and Japan. In many EU countries there is not only disinflation, but there is also deflation and a threat of a deflationary spiral. The impact on output, inflation rates and interest rate is studied with the GVAR model using a large macro data set for the EU, US, UK, and Japan. For the data set, see Binder and Gross (2014).

Figure 2: GVAR results - Deflation shock on GDP



[13]Data on those variables can directly obtain within the GVAR program.

Figure 2 shows the response of GDP to a deflationary shock in the Euro area, UK, Japan, and the US. It clearly reveals that there is much more deflationary pressure in the EU than in Japan, US and United Kingdom. Though many economists thought that Japan went through a period of deflationary pressure, it seems to look even more severe for the EU now.

3 Vulnerability through debt-deflation

The basic model that serves as a starting point for our theoretical considerations is described in appendix 2. In this first section, we want to start by introducing price level effects into our macro dynamics with leveraging. In the next section, we will introduce financial market reactions, and macro feedback loops, resulting from higher leveraging of the agents in the economy.

As mentioned, after the Great Recession researchers where wondering why the inflation rate did not quickly drop but moved down only very moderately. This is in contrast to the Great Depression, where the price level dropped by roughly 25 percent.[14] To understand the recent inflation persistence, we introduce inflation and price level dynamics into our basic model.

3.1 A model with debt dynamics and inflation

We follow the recent literature on slow inflation dynamics by letting the change of the inflation rate be driven by a slightly modified new type of a Phillips curve, such as in Werning (2012) and in Gross and Semmler (2015). We thus augment the basic model of appendix 2 with inflation and price level dynamics and incorporate their impact on the evolution of debt. The augmented core model then reads as follows:

$$\max_{C_t,\mathcal{I}_t,\mathcal{V}_t} \int_0^N U\left(C_t, N_t\right) e^{-\rho t} = \int_0^N \left\{ \frac{C_t^{1-\eta}}{1-\eta} - eN_t^{\chi} \right\} e^{-\rho t}$$

s.t.

$$\dot{N}_t = m^L\left(s_t \cdot \mathcal{U}_t, \mathcal{V}_t\right) - \sigma N_t \tag{1}$$

$$\dot{K}_t = m^B\left(\mathcal{I}_t/P_t, \mathcal{B}_t/P_t\right) - \delta K_t \tag{2}$$

$$\dot{d}_t = rd_t - \frac{1}{P_t}(\upsilon[P_t Y_t\left(K_t, AN_t\right) - \mathcal{F}_t - \Phi\left(s_t\right)\left(1-N_t\right) - \zeta \cdot \mathcal{V}_t - \kappa \cdot \mathcal{B}_t]) - \pi_t d_t \tag{3}$$

Here, C_t represents aggregate consumption, Y_t: aggregate production, A : (exogenous) labor productivity, $\mathcal{F}_t$: available financial funds through savings and external borrowing, $\mathcal{B}_t = \mathcal{F}_t - C_t$: offered bonds to firms, r_t : the interest rate, N_t: employment and $\mathcal{V}_t$: vacancies. In addition to costly search on the labor market, issuing bonds adds another cost factor to the macroeconomic resource constraint, with per-period flow costs for unmatched bonds measured by κ. The preferences are over consumption flows, C_t, and employment, N_t. The dynamics of eq. (1) represent the evolution of employment which is normalized to one. Eq. (2) denotes the evolution of the capital stock and eq. (3) represents the dynamics of aggregate debt in real terms (for both households and firms).[15] Our debt dynamics is written in a standard way if one allows for borrowing of the private (or public sector), possibly also from abroad.[16] Moreover, we have deflated the nominal variables with price level P_t, also the debt taken on, and we have to add the term $\pi_t d_t$ with π_t the inflation rate.[17]

In eq. 3, the term [.] represents external borrowing (> 0) or repayment (< 0), in the former case used for excess spending over domestic income. Moreover, we assume $\mathcal{F}_t = \mu C_t$, $\mu > 1$. Thus, consumption can be smoothed intertemporally, but investment funds might be restricted.

[14] For the price level fall, see Marglin (2007), and for the impact of price level fall and income fall on credit risk and bank defaults, see Bernanke (1983), but also Fisher (1933).

[15] We could also allow for sovereign debt here, though we do not specify what fraction of debt is driven by households, firms or the public sector.

[16] see Blanchard and Fischer (1989, ch. 2) and Blanchard (1986).

[17] We hereby assume that the capital stock in (2) is already deflated.

This means investment is more scrutinized through decentralized financial market matching mechanisms,[18] but if there is a consumption boom, more investment funds will also be available on the credit market.[19]

Moreover, search costs, $\Phi(s)$, are assumed to be fixed, with constant search effort $s = 0.2$. The function $m^L(s \cdot \mathcal{U}_t, \mathcal{V}_t)$ in eq. (1) is a a decentralized matching function on the labor market. Given the decentralized matching process $m^L(s \cdot \mathcal{U}_t, \mathcal{V}_t)$ the job finding rate of the unemployed will be $m^L(s_t \cdot \mathcal{U}_t, \mathcal{V}_t)/\mathcal{U}_t$ which will depend, fixing the search intensity, on the vacancies posted by firms and the unemployment rate.[20] The job finding rate is thus the ratio of the numbers of new hires divided by the number of workers searching for jobs. With higher unemployment and lower vacancies the job finding rate is lower.[21]

On the credit market there is also a decentralized matching mechanisms defined by $m^B(\mathcal{I}_t, \mathcal{B}_t)$, which represents the decentralized matching mechanisms, as a matching function for the credit market. Both matching functions for labor and credit markets display constant returns to scale and are represented by a Cobb-Douglas functions with exponents q_0=0.5 and $q_1 = 0.5$. The parameters σ, δ are the separation rate and depreciation rate of capital, and υ is our regime switching parameter which will be either 1 or 0, depending on the degree of permitted leverage of the economy.

As mentioned, in order to avoid a fourth decision variable, we have made the supply of funds for firms investments a function of the total supply of funds. Given then the external funding and the consumption decision, a fraction of funds can be used for providing bond offering to be matched with the bond demand arising from firms' desired investment $\mathcal{I}_t$. Funding for consumption will be available from domestic and external sources, but investment funding will be obtained on the credit market by the decentralized matching process on the credit market. Note that in this first step we do not have constraints on consumption smoothing.

Thus as in the basic model, here we assume that consumption is a direct decision variable and investment is expressed as intended investment, $\mathcal{I}$, to be matched with the supply of bonds, the supply of funds for bonds given by $\mathcal{F}_t - C_t$. As mentioned, this might be a reasonable assumption that allows us to work with a lower dimensional system. It also means that the screening and monitoring of investment funding takes place more extensively than funding for optimal consumption. In this context here, consumption is only indirectly constrained, namely through the state variables.

Finally, we have to formulate how we obtain the inflation rate and construct the price level P_t. Similar to some New Keynesian literature we assume that inflation rate and the price level adjust slowly.[22] As in the NK view we can then proxy the inflation dynamics by the output gap and a proxy for an expected inflation rate. Here, however, we are working with the rate of change of the inflation rate, derived from $\pi_t = \dot{P}_t$.[23]

$$\dot{\pi}_t = \beta(\frac{Y_t}{Y^*} - 1) + \eta^c{}_t \tag{4}$$

We let the change of the inflation rate respond to the output gap and some expectation term. Using eq. (4) we rely on demand and cost pressure arguments but we have chosen, because of numerical reasons, a short-cut of the Phillips curve. The inflation adjustment eq. (4)[24] follows in

[18] This in principle allows us to study more properly the heterogeneity of the euro area credit market.

[19] This for example, was likely to be the situation in Spain before the financial meltdown of the years 2007-9. Of course, there are likely to be constraints for households' borrowing as well, which will be discussed in section 4. For a more general empirical result of the dominance on household behavior in borrowing, see IMF (2015)

[20] This gives rise to the usual Beveridge curve.

[21] For details, including also time varying labor market participation rates, see Christiano et al (2014).

[22] Usually the Calvo price setting procedure or the Mankiw quadratic adjustment cost of prices are employed to get sticky prices. Our subsequent formulation is not inconsistent with views that presume that prices are driven by marginal cost and expected inflation rates, see Keen and Wang (2005). There is then shown that through a linearization the usual Phillips curve relationship is then proxied through a local output gap and expected inflation rate.

[23] See also Blanchard and Johnson (2013).

[24] Flaschel et al (2007) write the inflation rate being determined by $\pi = \beta_u(u^* - u) + \beta_Y(\frac{Y}{Y^*} - 1) + \kappa\pi^c$; with $\dot{\pi}^c = \beta_{\pi^c}(\pi - \pi^c)$. In the first equation, the first term on the right hand side defines the unemployment gap

principle Rudebusch and Svensson (2002; RS) as employed in Werning (2012).[25] In RS, however, inflation responds in a discrete time manner to time lags of the endogenous variables, to the output gap and a moving average of inflation rate, with the latter term proxying expected inflation. In our case, we have formulated a model corresponding to Rudebusch and Svensson, but written in continuous time, using the rate of inflation as a differential equation. Note that starting with the derivative of the inflation rate might make some sense, since the inflation rate does not jump and in many EU countries still tends to be slightly positive though the change of the inflation rate itself was negative for a long time and now the inflation rate slowly turns to be negative too.

The inflation rate expression, ${\eta^c}_t$, represents some inflationary climate – of the change of inflation rates – in which the current inflation dynamics is operating. The climate variable ${\eta^c}_t$, is thus a magnitude that is related to some medium run and can be viewed to be updated in an adaptive fashion, as explained in the footnote for eq. (4). Our inflation climate variable is constructed in a similar way as in Ball and Mazumder (2014) who introduce some smooth process of inflation expectations by anchoring the inflation expectations in survey data.

Note that our inflation dynamics could be interpreted as based on cost push pressures and demand pressures,[26] affected by the output gap and thus capacity utilization. Note also that in eq. (4) we only use goods' price inflation and thus assume that wage and price inflation do not differ much when averaged over the medium-run. Wage cost pressure that firms are facing could be formulated in a second term. For a detailed analysis of the stability properties of such price and wage Phillips curves, see Flaschel et al. (2007). Empirical evidence on slowly moving inflation rates, justifying to focus on the change of inflation rate, as in eq. (4) is given in Gross and Semmler (2015) and in appendix 5, where a regime dependent Phillips curve is studied.

An important reason why there is disinflation rather than deflation – or the inflation does not become strongly negative as much research recently has pointed out – is that in contractions, such as the recent one, the demand pressure will reduce prices but the risk premia and credit spreads increase credit costs, in particular for credit on working capital, pushing up costs.[27] So there is a cost push as well as a demand effect working, preventing the inflation rate from falling less than one would expect.

Lastly we need to introduce the aggregate price level dynamics, since this is used in eqs. (2)-(3). The price level dynamics can be defined through the following differential equation:

$$\dot{P}_t = \pi_t \tag{5}$$

Note that eq. (5) can be used to determine a price index, starting with $P_0 = 1$, that represents the integrated inflation rates as a solution of eq. (5), so as to obtain P_t.

The following parameters for the NMPC solutions are used: $\mu = 1.3$ and $\beta = 035$, $\kappa = 0.1$, $\rho = r = 0.03$, $\delta = 0.03$, $\sigma = 0.04$, $\alpha = 0.36$, $A = 1$, $\xi = 0.07$, $\chi = 5$, $e = 1$. The parameter υ is set to one, which means there are no credit constraints (if set to zero there are credit constraints). In our numerical solution algorithm we start with a price level $P_0 = 1$, integrate the inflation rates following eq. (5) to obtain P_t and deflate appropriately the nominal variables such as the demand for firms' funds and the bond supply, $\mathcal{I}_t/P_t$, $\mathcal{B}_t/P_t$, in eq.(2). We also deflate debt service in eq.

(representing pressure from the labor market) and the second the output gap (representing pressure from the product market), see Flaschel et al (2007). The second equation, a differential equation, defines an expectational term, the inflation climate, with a path toward a steady state inflation rate. The later equation represents the change of the climate inflation and can be interpreted as in Ball and Mazumder (2014) as an anchor of inflation rates, as SPF's consensus forecast of CPI inflation or as inflation climate, as in Flaschel et al. (2007), see also Gross and Semmler (2015) and their use of survey data. In our eq. (4) we have only used the first term of the inflation dynamics π_t, and we have set $\eta_t^c = \dot{\pi}^c$. In our subsequent numerical solutions we use eq. (4) for an inflation dynamics, and thus employ only the output gap and let the dynamics of $\dot{\pi}^c$, being generated by some moving average of the change of inflation rates.

[25] Werning (2012), however assumes a form where the current change of the inflation rate is anchored in the purely forward looking form of the agents' behavior, and thus he has a negative sign for the output gap. Econometric evidence for this formulation seems to be very weak as Gordon (2010) and Ball and Mazumder (2014) argue. This criticism does not hold for Rudebusch and Svensson (2002). We follow more the latter approach.

[26] see also Christiano et al. (2014).

[27] For details, see Christiano et al (2014)

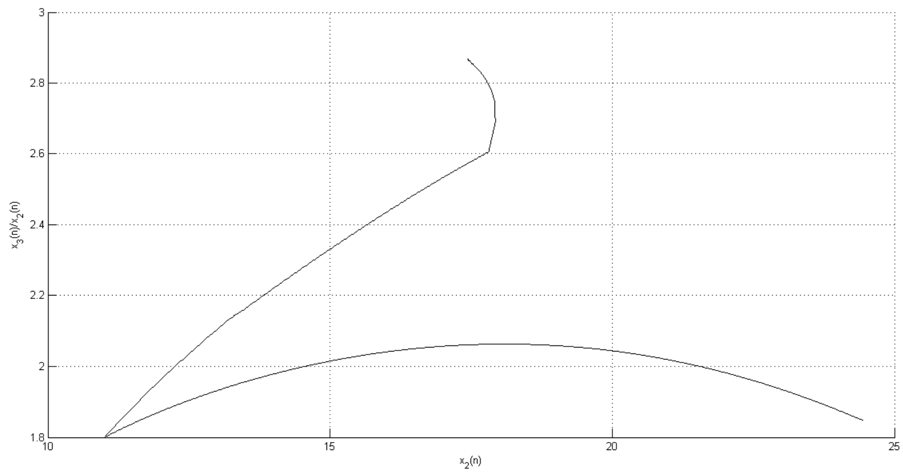


Figure 3: Solution path of capital and real debt to capital ratio; initial condition for $N(0) = 0.9$ and $D(0)/k(0) = 1.8$, lower trajectory: because of low and declining real interest rate, k_t moves up and the leveraging ratio moves up a bit and then down; upper trajectory: because of deflation and high real interest rates, and high debt, k_t first moves up, but then down, and debt and the leverage ratio become unstable

(3), income, total financial funds, the search cost for jobs as well as the search cost for issuing bonds. This way we obtain the evolution of the variables in real terms, and also debt in real terms, as determined by the level of debt, the excess of income over spending and the interest rate and inflation rate.

We solve this higher dimensional macro model (1) - (5) using NMPC with the above defined objective function for a finite time horizon. In our numerical solution we start now in the vicinity of some steady state of the basic model – but, to proxy a recession, we also start with a negative output and employment gap. We take $N(0) = 0.9$ as the initial employment rate.[28] We track the path of all state variables, including the inflation and price dynamics as defined in eqs. (4) and (5).

In the numerical solution of the model (1)- (5) we can distinguish two cases, see figure 3. The upper trajectory represents one case, the lower trajectory another case. As our price adjustment process suggests sticky prices we have slow inflation rate movements. Moreover, we assume, when the economy contracts, that we have different initial conditions for the price adjustment process. How do those two cases emerge in figure 3? This will be discussed next.

3.2 Debt deflation and slow recovery

We commence with the lower trajectory of figure 3. The lower trajectory of figure 3 presumes that we start with an economy that had not experienced disinflation, the inflation rate is at target, roughly 2 percent, we thus commence with an initial $\pi(0) = 0.02$. Yet, the fiscal or monetary policy may have initiated slight recoveries, and the output gap starts closing again. Given a nominal interest rate of $r = 0.03$,[29] we can observe that the ratio of real debt to capital stock first rises but then declines after a while. The latter is due to low interest rates, excess of income over spending, positive inflation rates, and a rising price level, arising from eq. (4), and thus declining real interest rates. As we can observe from figure 4 and 5 the inflation rate goes up to almost

[28] In our model, there is no labor force inactivity, so the employment rate is simply the inverse of the unemployment rate.

[29] Note that in this section we keep the nominal interest rate on a fixed level, but of course the central bank may reduce the interest rate to the lowest bound possible. On the other hand market credit cost may still be higher, due to credit spreads. To capture this effect we keep a constant interest rate, this will be changed in sect. 4.

$\pi_t = 0.045$ and the employment gap, figure 5, declines (and the output gap declines as well, not shown here).[30]

Note that we have imposed an initial inflation rate of $\pi(0) = 0.02$, which was roughly the inflation target of the central banks, at the beginning of the Great Recession. We start with this target, even if the economy moves into a recession with a negative output gap. Yet, prices are sticky downwards. There is a positive inflation rate, but it is first slightly decreasing, see figure 4, so the inflation rate first moves slightly down with excess capacity and a negative employment gap, both representing downward demand pressures, but then rises again. This may in fact represent some of the countries experience when there was no self-enforcing debt deflation or disinflation, for example the US, and the UK, but also Germany, and France, where inflation rates have not fallen much.

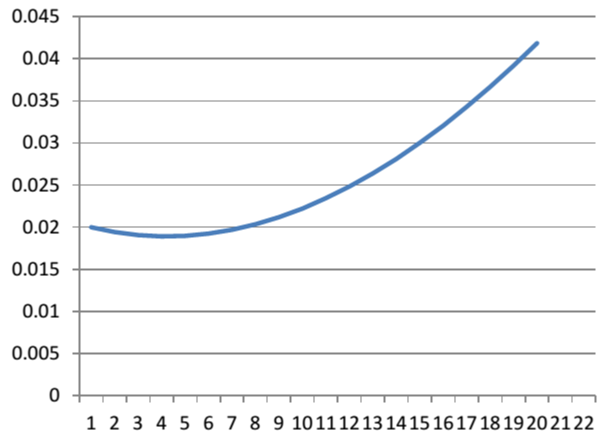


Figure 4: Inflation rate corresponding to lower trajectory of figure 3; starting with negative employment gap, first slightly declining inflation rate, then inflation rate rising with diminishing employment gap, for initial $\pi(0) = 0.02$

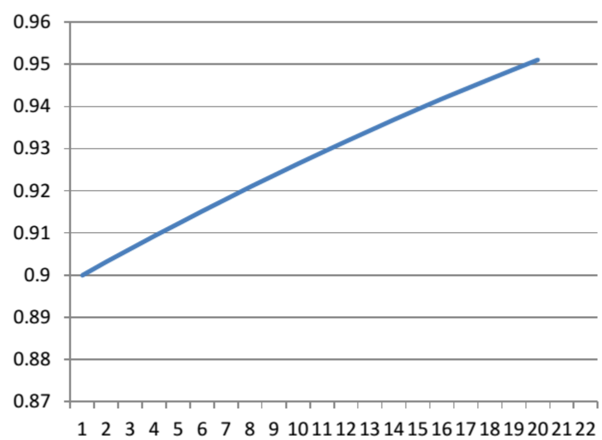


Figure 5: Employment gap corresponding to lower trajectory of figure 3; diminishing employment gap over time, for initial values $\pi(0) = 0.02$

[30]It is interesting to observe the employment gap and inflation rate. They are both driven by the output gap which is endogenously given from the system (1)-(5)not depicted here. Figures 4 and 5 represent the inflation path given by eq. (4) and the employment gap path respectively. As observable, the inflation rate is rather sticky, it is not necessarily negative with a negative employment gap, see figure 4, but moves only down little with the existence of an employment (output) gap. We thus can observe disinflation with such a gap, but not necessarily a deflationary process.

The main effect of the path toward sustainable debt comes from the growth or income term in eq. (3). The nominal interest rate, as cost of debt service is almost canceled out by the inflation rate but if there is sufficient income growth the term $\upsilon[.]$ may have have a positive sign, allowing the economy to grow out of debt. So, output and income growth also allows the output gap to close, and with this the inflation rate is rising, see figure 4 where the inflation rate is moving up to 4.5 %, reducing even more and more the real cost of borrowing.[31]

3.3 Debt deflation and prolonged recession

Next we discuss initial conditions for inflation rates to be much lower. We start now with a zero initial inflation rate, thus with $\pi(0) = 0.0$, triggered by a negative output gap. In some sense we assume the central bank cannot - or is not willing– to generate lower interest rates fast enough, so that the inflation rate slides down, and may even become negative, as discussed in sect. 2. Since the inflation rate is sticky through eq (4), even if there is a slight recovery, the inflation rate is only rising slowly with declining output and employment gaps.

In figure 3, the upper trajectory, shows the path of the real debt to capital ratio when we start with an economy that had experienced disinflation and the actual inflation rate has become zero, or negative. We start with an initial $\pi(0) = 0$, but, because of high real real interest rates (nominal interest rate is 0.03 and inflation rate zero), the slightly rising inflation rate reduces the nominal interest rate only very little – and the high debt levels moves up further. The last term in eq. (3) is very small as compared to the first term, and if the middle term does not move much, because output and income does not grow, debt will rise.[32] We can observe here that the debt and leverage ratio eventually become unstable, though the capital stock, k_t first moves up, and declines afterwards.[33] The main effect on the rise of debt comes from both, the rise of the real interest rate and the slow or negative income growth.[34]

Figure 6 is for the upper trajectory of figure 3, depicting the path of the inflation rate. Figure 7 shows the same closing of the employment gap which is driven by the closing output gap. But note that all of this still holds if we stay in a regime with little financial stress, with no rise in credit spreads, but below the steady state variable of $N_t < N^*$ and $Y_t < Y^*$ (see figure 7 for the employment gap). This may in fact represent some EU countries experience when there was self-enforcing debt deflation or disinflation, for example Greece, Portugal, and Spain, and also Italy recently, where inflation rates moved down to zero and even below zero, letting real interest rates rise.[35]

Tough in both cases, the upper and lower trajectory in figure 3, the economy may recover but in the case of the lower trajectory, the economy recovers more quickly whereas for the upper trajectory, with only slowly moving inflation, rising real interest rate and possibly rising debt burden, there is further increase of debt overhang and the economy is possibly moving into a slow moving debt crisis to be discussed further in sect. 4.

Note that in figures 4 to 7 the corresponding output and employment gaps, and thus the inflation rate, are driven by the system (1)-(5). In our simulations a normal level of employment is set at 95 % of the available labor force. We have normalized this to 1 in figure 7. The employment level, given the large output gap, we start with, is low and thus unemployment is high, roughly about 10 percent, as one could observe after 2007-8 in most countries. But also note that the employment gap is endogenously generated, and it is also a result of the vacancy rates chosen by firms, given the dynamic model (1)-(5).

Yet, as mentioned before so far we have presumed no further financial market reactions and financial market stress, and we have not taken into account a possible regime change on the financial

[31] This is what Tobin (1998) had in mind when discussing debt sustainability. Note that this is also consistent with the debt sustainability defined by Bohn (2007).

[32] An argument used in the study of sustainable debt already by Tobin (1998). This maybe amplified if financial stress jumps up.

[33] If the inflation rate it was falling recently further, it was presumably due to the decrease in the oil price.

[34] This again can be illustrated by using the Tobin debt sustainability model, see also Fisher (1933).

[35] But note if the bonds and thus the debt are partially indexed, one would get a weaker effect, as will be discussed in section 4.

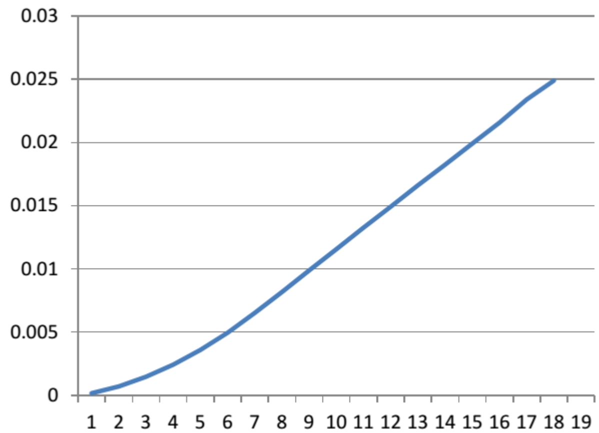


Figure 6: Inflation rate corresponding to upper trajectory of figure 3, starting with a zero initial inflation rate, triggered by a negative output gap, inflation rate only slowly rising with declining output and employment gaps, initial inflation rate $\pi(0) = 0.0$

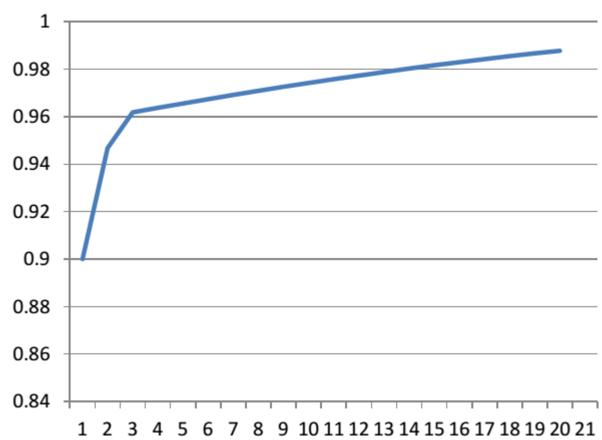


Figure 7: Employment gap corresponding to upper trajectory of figure 3, first faster then slowly declining employment gap

market, with rising default risk and credit spreads, and possibly adverse macro feedback loops. One is likely to expect different outcomes with such a financial meltdown. We will explore some important empirical features next and other aspects in appendix 5.

4 Financial market instability and regime change

We want to allow now for a rise of financial market stress, possibly due to rise of unsustainable real debt and rise of default risk (of households, firms banks, sovereign), resulting in higher default risk, risk premia and credit spread and thus in a contraction of credit flows, as indicated in sect. 2. We will thus introduce endogenous rise of risk premia and credit spreads to become strong factors in the macro dynamics which also trigger credit cost to go beyond the real interest rate effect discussed in the previous section. To study this issue we can set the price index $P = 1$ and thus $\pi = 0$, for all t. Since we want to consider another major contractionary force now, the Fisher effect is neglected in the next step. This also can be seen as also roughly equivalent to fully indexed bonds and thus debt.[36]

We again refer back to the basic model of appendix 2, but now we will introduce endogenous credit cost and and credit spreads. We explore two model variants. In one version there are only very weak macro feedback effects to aggregate demand when credit spread is rising. In the other version we will include stronger macro feedback effects. This modeling strategy is important because it brings out the role of leveraging, debt overhang and financial market and credit cost jumps which are not present in the liquidity model such as Werning (2012).

4.1 Endogenous credit spreads and weak macro feedbacks

Our model with endogenous risk premia and credit spreads[37] can be written as follows.

$$\max_{C_t, I_t, \mathcal{V}_t} \int_0^N e^{-rt} U(C_t, N_t) dt \tag{6}$$

s.t.

$$\dot{N}_t = m^L(sU_t, \mathcal{V}_t) - \sigma N_t \tag{7}$$

$$\dot{K}_t = m^B\left(\mathcal{I}_t, \mathcal{B}_t\right) - \delta K_t \tag{8}$$

$$\dot{D}_t = r(fs_t|\gamma,)D_t - \upsilon[Y_t - C_t - g_t K_t - \Phi\left(s_t\right)\left(1 - N_t\right) - \zeta \cdot \mathcal{V}_t - \varphi(g_t K_t)] \tag{9}$$

In eq. (6) there are again preferences over log utility and non-working (leisure) time. The policy variables are consumption, growth rate of capital stock, and vacancies posted by firms, $C_t, g_t, \mathcal{V}_t$.[38] Eq. (7) again represents the decentralized matching mechanism on the labor market and credit market. Again the search effort s will be taken as constant. By the aid of the decentralized search and matching on the credit market, in eq. (8) the capital stock increases due to investment but declines due to a capital depreciation rate δ. The interest payment on debt, $r(\cdot)D_t$, now increases with debt but the surplus $\upsilon[Y_t - C_t - I_t - g_t K_t....]$ – negative excess absorption – decreases debt. Hereby again we have set $I_t = g_t K_t$. The interest rate is now driven by financial stress, fs_t, as discussed above.

The expression in brackets $\upsilon[\cdot]$ can be interpreted again as change in external liabilities. Thus, here again, since consumption and investment are separate policy variables we allow for external

[36] This might avoid the criticism of the EK model of assuming not having indexed bonds.

[37] Blanchard (2013) expresses this as a jump to a second, bad, equilibrium: "The higher the debt, the higher the probability of default, the higher the spread on government bonds, and the harder it is for the government to achieve debt sustainability. But the adverse effects do not stop there... ".

[38] Actually in the numerics we can take $c_t = C_t/k_t$, so that the first two choice variables can be confined to reasonable constraints between 0 and 1.

borrowing. As before, $\varphi(g_t k_t)$ is a quadratic adjustment cost for investment, included in eq. (9). Overall the model has now three decision variables and three state variables.[39]

Arellano (2008) argues that with probability of default the market price of bonds is affected, to be paid for the next period's bonds. If the bond defaults the value of the bond is zero, there is a welfare loss for the bond issuer.[40] If the bond issuer is not defaulting the bond continues to be in use and the bond price (inversely the yield) will fall due to the level of debt and rise due to a positive shock on income. Now one can think of those two extreme scenarios as off-on cases.[41] If we want to smooth out the off-on cases, as the only two scenarios, we can perceive a continuum of cases where the probability of default may steadily rise starting from a low level, and then leveling off, where no bonds can be be issued any more. One can make the bond prices and thus the yields, a nonlinear function of financial stress and leveraging.[42]

Thus, overall, in contrast to the model variant of section 3, we could assume that the bond yields are a nonlinear function of some measure of leveraging, the debt to capital stock ratio. We may represent $r(fs_t|\gamma, c))$ by a logistic function of the following type[43]

$$r(fs_t|\gamma, c) = [1 + \exp(-\gamma(fs_t - c))]^{-1},\ \gamma > 0 \tag{10}$$

with the credit spread and interest $r(\cdot)$ arising from financial stress, fs_t, in particular if it is rising above some threshold, c. For the construction of the financial stress index, see Schleer and Semmler (2013). This function represents roughly the function that has been observed by de Grauwe (2012) in EU data[44] and can be thought of representing the shape of financial stress as has been observed in Schleer and Semmler (2013). The interest payment on bonds rises with the shape of the function of eq. (10), first slowly, then more rapidly but is then finally bounded.

One would expect that with less financial stress and lower interest payments on bonds a higher steady state leveraging ratio is admissible. Again, debt is sustainable if the second term in eq. (9), the excess of income over spending, is equal to the first term, the interest payments on debt.[45]

The result of our finite horizon model as presented in eqs. (6)-(10), using our NMPC methodology, is shown in figure 8. Note that in the simulations the upper part of the trajectory is unstable and the debt to capital ratio eventually becomes unbounded.

[39]Note that we assume here a difference of interest and discount rates. Eggertsson and Krugman (2011), and also Brunnermeier and Sannikov (2014), give a justification of why the interest rate might be different from the discount rate.

[40]See also Roch and Uhlig (2011). They include a utility loss in the welfare function which is similar to the Blanchard (1986) model and in Mittnik and Semmler (2014).

[41]Blanchard (2013) expresses this as multiple equilibrium dynamics: "The higher the debt, the higher the probability of default, the higher the spread on government bonds, and the harder it is for the government to achieve debt sustainability. But the adverse effects do not stop there. Higher sovereign spreads affect private lending spreads, and in turn affect investment and consumption. Higher uncertainty about debt sustainability, and accordingly about future inflation and future taxation, affects all decisions. I am struck at how limited our understanding is of these channels. Reduced form regressions of growth on debt can take us only so far. ....At high levels of debt, there may well be two equilibria, a "good equilibrium" at which rates are low and debt is sustainable, and a "bad equilibrium" in which rates are high, and, as a result, the interest burden is higher, and, in turn, the probability of default is higher. When debt is very high, it may not take much of a change of heart by investors to move from the good to the bad equilibrium." Blanchard (2013:3).

[42]As empirics has shown, financial stress is related, to great extent, the high leveraging and bond yields, but expresses many more factors than leveraging and bond yields, see Schleer and Semmler (2014). But as Principle Component Analysis shows there, bond yields and a proxy for leveraging, are strong components in the PCA. Others, for example, Gilchrist et al (2011), have just added a persistent shock to the leverage ratio to obtain a higher bond yields and thus credit spreads.

[43]The following function has a similar shape as the exponential function used in the Smooth Transition Regression models, see sect. 5, where a transition variable, and its move through a threshold, changes the dynamics of the state variables, see Schleer and Semmler (2013). We approximate here the above function by an arctan function which has the same shape, we use

$$r(fs_t|\gamma, c) = \beta \cdot \arctan(D_t/K_t).$$

This function is numerically more convenient when we use our NMPC method.

[44]Representing there EU sovereign debt and bond yield data which however have also been observed for bond yields in the private sector, see Blanchard (2013). Others have formulated this as the high and low probability of default: In our case however now stylized as smooth transition from low to high probability of default and its impact on credit spread.

[45]Again, sustainability is used here in the sense that the debt to capital stock ratio converges to a constant.

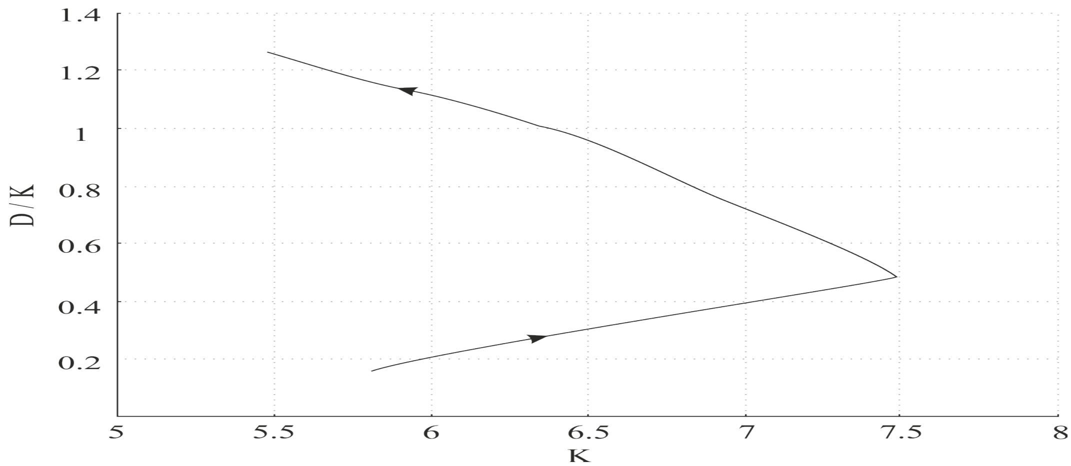


Figure 8: Dynamics with credit spread, without macro feedback loops

To obtain figure 8, we have set the macro feedback effects to be weak.[46] As to the solution path for the capital stock and leveraging, figure 8 shows the lower interest payment on bonds first admits a higher capital stock and higher leveraging. Yet, as the interest rates – in our case the risky bond yields– reach a certain threshold, we observe that with an increasing leveraging and credit risk and risk premia, capital stock stops rising but the leverage ratio is rising further. This is occurring when the credit spread is moving beyond a certain threshold. Here then finally there is unsustainable debt since the interest payment becomes higher than the surplus to service the debt, as the eq. (9) indicates.[47] The contraction in output, investment and capital stock, and the rise of debt, as they are occurring at the turning point in figure 8, may also be impacted by macroeconomic feedback effects as they are discussed next.

4.2 Endogenous credit spreads and strong macro feedbacks

Next we not only allow for risk premia and credit spreads to be endogenous, but also for a feedback effect of leveraging and bond yields on investment and consumption decisions, thus on aggregate demand and output, so that one might move into a slow moving debt crisis.[48] Those macro feedback effects can also be interpreted as rise of macro uncertainty and its effect on investment and consumption as in Cesa-Bianchi et al. (2014). Our model is now:

$$\max_{C_t, I_t, \mathcal{V}_t} \int_0^N e^{-rt} U(C_t, N_t) dt \tag{11}$$

s.t.

$$\dot{N}_t = m^L(s_t U_t, \mathcal{V}_t) - \sigma N_t \tag{12}$$

$$\dot{K}_t = m^B(\mathcal{I}_t, \mathcal{B}_t) - \delta K_t \tag{13}$$

$$\dot{D}_t = r(fs_t|\gamma, c))D_t - v[Y_t^a - C_t^a - (g_t K_t)^a - \Phi(s_t)(1 - N_t) - \zeta \cdot \mathcal{V}_t - \varphi(g_t K_t)] \tag{14}$$

[46]Note that the credit spread is nevertheless rising due to financial market stress. Others, see Werning (2012) and also Gavin et al (2013) have allowed the interest rate to rise solely as a result of the central bank's monetary policy rule. They do not have credit and financial market in their model. In our case the central bank can change the policy rate, but what appears to be more important is the financial market stress and credit spread. They also do not consider the impact of the interest rate on debt sustainability.

[47]This maybe be magnified by the reversion of the effect as mentioned before: namely the risk and risk premia rising, discount rates rising and falling (or negative) capital gains, not supporting the debt repayments any more. So debt would rise faster.

[48]Blanchard (2013) supports such a statement by referring to adverse macroeconomic feedback effects, see footnote 39.

The difference to the model variant of sect. 4.1 above, again with $\upsilon = 1$, is here now that the credit spreads maybe a nonlinear function of the debt to capital stock ratio, as before, but there is also an endogenous effect of the credit spread on demand, output and income. Thus the major difference to the previous variant is now that macroeconomic conditions can worsen as a result of an endogenous effect of credit spread on aggregate demand and output.[49] There are indeed important macroeconomic feedback mechanisms that one often can observe in the data, for the US for example, see also Hall (2011) with respect to aggregate demand, Taylor (2013) concerning investment and employment, Christiano et. (2014) with respect to a large number of macro variables, and Blanchard (2013) with respect to sovereign debt risk and its spillover to private borrowing cost.

To specify the macro feedback loops, we can make the actual consumption and investment demand depending on credit spreads triggered by rising risk premia and yields of bonds. This would be given by:

$$C_t^a = f(r(fs_t|\gamma, c))C^{opt} \tag{15}$$

$$I_t^a = g(r(fs_t|\gamma, c))I^{opt} \tag{16}$$

Though optimal consumption and investment plans are chosen over the planning horizon N, actual consumption and investment declines due to rising risk premia, credit spreads and possibly financial stress.[50] So, overall we may have:

$$Y_t^a = u(r(fs_t|\gamma, c)Y_t^{opt} \tag{17}$$

We can take

$$u(r(fs_t|\gamma, c)) = (1 - r(fs_t|\gamma, c)) \tag{18}$$

and use the rising credit spread as self-enforcing mechanism reducing demand and output. We thus could also write:

$$Y_t^a = u(\cdot){K_t}^{\alpha} \tag{19}$$

which now indicate a decreasing utilization of capital. Thus, if risk premia and credit spreads rise, and their are macroeconomic feedback effects to aggregate demand, this will reduce consumption and investment demand, actual output, Y^a accompanied by a lower utilization of capacity. Thus tax revenue, as well as the net income, to service the debt, may fall. This might make then credit and bond issuing unsustainable – generating a further jump in credit spread.[51]

Those economic outcomes seem to be less due to shocks but rather due to macroeconomic feedback loops and their adverse economic impact[52] which may arise because of the following:

- If the financial market goes into distress and asset prices fall, there is the wealth effect reducing aggregate demand and both consumption and investment demand are likely to fall

- The share of households that are income and credit constrained, in the sense of Gali et al (2008), and households that are higher leveraged and are under financial stress[53] are significantly rising in a contraction period of the business cycle, see also Mittnik and Semmler (2012a)

[49] As aforementioned, Blanchard explicitly refers to those macro feedback mechanisms that not only affect the sovereign debt sustainability bald also aggregate demand and output, Blanchard (2013:3).

[50] In the local linearization version of the New Keynesian, this would just show up in the rise of the interest rate in the output equation, see Werning (2012).

[51] Yet one might also face insolvencies of households, firms or financial intermediaries, in the period of high financial stress, as discussed in Semmler and Semmler (2013), which would amplify the above described contraction.

[52] A systematic study of macroeconomic feedback effect, know from the history of macroeconomics, partly stabilizing partly destabilizing, are extensively discussed in Charpe et al (2013)

[53] The share of those households matter, since there is empirical evidence that the drop in demand will be larger for households with larger debt and that are forced to deleverage more, see Eggertsson and Krugman (2011).

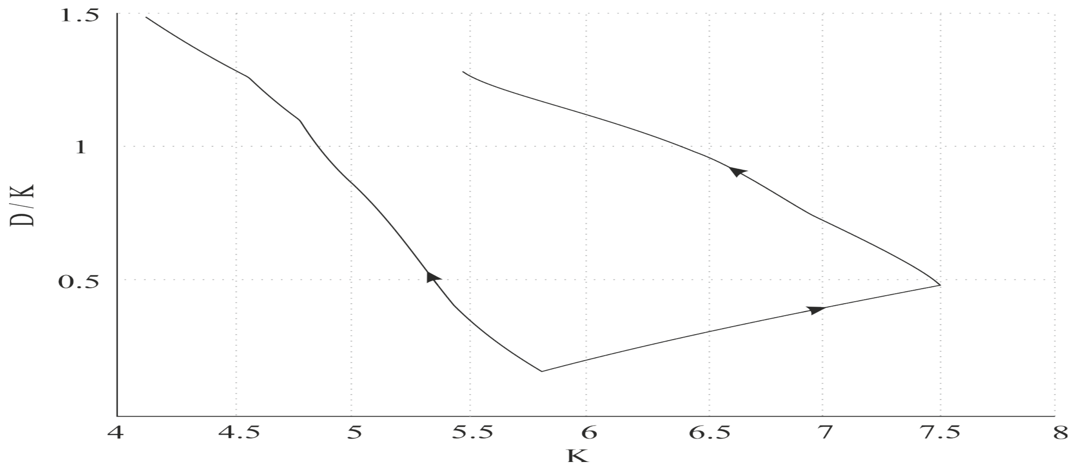


Figure 9: Debt dynamics with endogenous credit spreads and weak macro feedback loops (right trajectory), and debt dynamics with endogenous credit spreads and strong macro feedback loops (left trajectory), both starting from the initial condition $K(0) = 5.8$, $D(0) = 1.2.$, both trajectories indicating instability.

- As the financial market forces trigger financial stress,[54] the central bank may have no instruments available – or is not willing – to force the interest rate down further and/or to reduce risk premia and credit spreads, for example by purchasing sovereign bonds to drive down sovereign risk and risky bond yields[55]
- Given the labor market constraints, a fraction of private households could start strongly deleveraging which reduces income and liquidity of other households and firms, which might be accompanied by debt deflation spiral that Eggertsson and Krugman describe (2011)[56]
- Finally, there could occur even a worse feedback: a weak financial sector, holding risky sovereign debt, may come under severe stress, because sovereign bonds may go into default and banks reduce lending to the real economy, or worse, may even default[57]

Whereas the first three destabilizing mechanisms have been known in the literature and are often viewed to generate a vicious cycle, the last one, which has recently been discussed, adds a more dangerous mechanism which has been called "diabolic loop".[58]

With those stronger macroeconomic feedback loops, we can expect, starting with a certain debt to capital stock ratio that the above feedback mechanisms lead to negative wealth effects, higher financial market stress and higher default premia, higher credit spreads and lower output leading in turn to a contraction in the utilization of the capital stock, and capital stock itself, and to an increasing leveraging ratio.[59] Note that a situation is sketched here where the central bank is apparently unable or incapable to bring down sufficiently the risk premia, credit spreads and financial stress, through asset market interventions.

Figure 9 shows two solution paths for our system (11)-(19) using our NMPC procedure: the right trajectory is without (or very weak) macro feedback effects on consumption and investment

[54]See Schleer and Semmler (2013) for a banking oriented stress index.

[55]The ECB in Europe was initially very constrained by the Maastricht Treaty not to purchase sovereign bonds. Later this was relaxed by allowing it to purchase sovereign bonds on the secondary market, though there were number of programs that by-passed the Maastricht Treaty, as the recent ECB QE program with extensive bond purchases that has brought down significantly the credit spread, see sect. 4.3.

[56]A detailed discussion of further macroeconomic feedback effects of this type can be found in Charpe et al. (2013).

[57]See Brunnermeier and Oehmke (2012).

[58]See Brunnermeier and Oehmke (2013), and also Bolton et al (2011).

[59]This could equivalently create a downward spiral in net worth, if the model is written in terms of net worth, as in Brunnermeier and Sannikov (2014) and Stein (2012).

demand, and the left trajectory is the solution path under the impact of strong macroeconomic feedback effects. For comparison we have include in figure 9 the solution of shown in the figure for the weak macro effects. The left trajectory of figure 9 represents the path when the macro feedback effects become strongly effective. As one can observe there is now a much stronger contractionary effect as compared to the dynamics shown in the right trajectory. Yet, both solution path of both trajectories indicate that macro variables diverge and debt will not be stabilized, and slow moving or acute debt crisis may occur.

5 Multi-Regime VAR estimations

Our main drivers of debt deflation - financial stress dynamics in the above approach are output, inflation rate, state dependent credit cost (interest rates) and a measure of regime defining financial stress. Our model variants would predict a tranquil regime, as presented in appendix 2, with little effects from shocks, and a financial stress regime with strong effects from shocks as demonstrated in sect. 4. Long time series data to estimate such a higher dimensional MRVAR are not easy to obtain, so we used some proxies, particularly for the credit cost.

We utilized three different sources for our dataset: change in GDP and the inflation rate are taken from the GVAR project (Smith and Galesi, 2014). Their data is taken from the International Financial Statistics (IFS), where GDP is a real index with base year 2005 and inflation rates represent changes in consumer prices.

State dependent credit cost would ideally be represented by private lending cost. However, for the period until 2003 data on private lending cost was not available. Therefore we detrended the long-term interest rate provided by Smith and Galesi (2014) (Interest Rates, Government Securities, Government Bonds concept) for the period until 2003. From 2003 until 2013 long-term cost of borrowing from the MFI interest rate statistics by the ECB[60] was used.

Our regime defining variable is financial stress. For this we take the ZEW FCI index, which acts as the endogenous threshold variable in our MRVAR model. It is taken from Schleer and Semmler (2014) and discussed in more detail in appendix 5.

5.1 Methodolgy

For our estimation we rely on a MRVAR with the FCI as an endogenous threshold variable, which allows for regime or state dependent effects of increasing financial stress.

We are using a nonlinear approach due to the shortcomings of linear VARs. In a linear model, with orthogonal impulse responses, state dependent effects of shocks are not taken into account, while impulse responses are symmetric with respect to the sign of the shock and linear in terms of their size (Koop et al. 1996). Thus, given our model in section 4, it would be inappropriate to use a linear approach. Instead we use a multi-regime model which allows us to study regime-dependent effects. The MRVAR can be defined the following way:

$$\mathbf{y_t} = \mathbf{c_i} + \sum_{j=1}^{p} A_{ij}\mathbf{y_{t-j}} + \epsilon_{it} \quad \text{if } \tau_{i-1} < r_{t-d} \leq \tau_i \tag{20}$$

where $\mathbf{y_t} = (y_{1t}, \ldots, y_{nt})$ represents the endogenous variables and $\mathbf{c_i}$ is a vector of regime-dependent constants. τ stands for the threshold values and r_{t-d} is the endogenous threshold variable, while d is the threshold delay.

Ultimately, like in the case of a linear VAR, we are interested in the effects of a shock to a specific endogenous variable on the equation system depicted above. However, orthogonal impulse responses are not appropriate here due to the shortcomings described above. Instead one has to use generalized impulse response functions (GIRF) (see Koop et al. 1996). GIRF allows us to take asymmetries with respect to the sign of the shock, the size of the shock and its history-dependence into account.

[60] https://sdw.ecb.europa.eu/browse.do?node=2018774

	90%	95%	97.5%	99%	Test Statistic	P-Value
Crit. Value MRVAR	43.86298	47.91352	51.34062	55.57802	70.05991	0.00200

Table 1: Spain: Test of linear VAR against MRVAR

	90%	95%	97.5%	99%	Test Statistic	P-Value
Crit. Value MRVAR	45.13078	49.10033	52.85407	59.14743	43.73376	0.12500

Table 2: Italy: Test of linear VAR against MRVAR

	90%	95%	97.5%	99%	Test Statistic	P-Value
Crit. Value MRVAR	42.87387	46.56264	53.05626	63.52801	51.52063	0.02700

Table 3: Germany: Test of linear VAR against MRVAR

	90%	95%	97.5%	99%	Test Statistic	P-Value
Crit. Value MRVAR	44.40238	48.20203	52.15612	57.37792	47.96587	0.05100

Table 4: France: Test of linear VAR against MRVAR

The GIRF work the following way:[61] we split our data set into subsets of observations according to the regimes they belong to and analyze each regime on its own by taking a random starting value from a given regime and simulating the model with bootstrapped (regime-specific) residuals. We repeat the simulation with the same starting values and bootstrapped residuals, but we add an additional shock to one variable in period one. This procedure is then undertaken $100 - times$ for a given starting value and randomly drawn residuals and afterwards the average of the simulations is computed. We repeat the simulation $300 - times$ for each regime where histories are drawn randomly for each of them.

5.2 Empirical Analysis

Before estimating a MRVAR we have to test for the significance of threshold effects, where we use the test developed by Lo and Zivot (2001) to test the null hypothesis of linearity against the alternative of threshold effects. This test can be seen as a necessary condition for the appropriateness of our theoretical and empirical models. If the test fails to reject the null hypothesis of linearity, state dependencies of our economies with respect to financial market conditions would not be observed and a linear VAR would be adequate.

The threshold test was conducted with 1000 bootstrap replications for each country and a trimming value of 0.1 which guarantees that each regime contains at least 10% of all observations. The results are shown in tables (1) through (4).

As can be seen from tables (1) through (4), the tests reject the null hypothesis of linearity for all countries, but Italy. However, for reasons of comparison, we still decided to estimate a MRVAR model for Italy.

As the test for threshold effects suggested estimating a model with two regimes, we conducted our analysis with the same settings as our threshold tests: the trimming value was set to 0.1, while the threshold value and threshold delay were identified by a grid search with the objective of minimizing the sum of squared residuals, where the threshold delay was set to one for all countries. The lag lengths of of our models were informed by the Schwartz criteria, which suggested a lag length of one for all countries.[62].

In the next step we employed GIRF to simulate the effects of a shock to the FCI index on the change in output. To check for robustness, a second version of the model was estimated, where the first difference of the interest rate was used instead of the level of the interest rate. The impulse responses are depicted in figures (10) through (13).

[61] A detailed algorithm for computing the generalized impulse responses is described in appendix 3
[62] The estimation results are listed in appendix 7

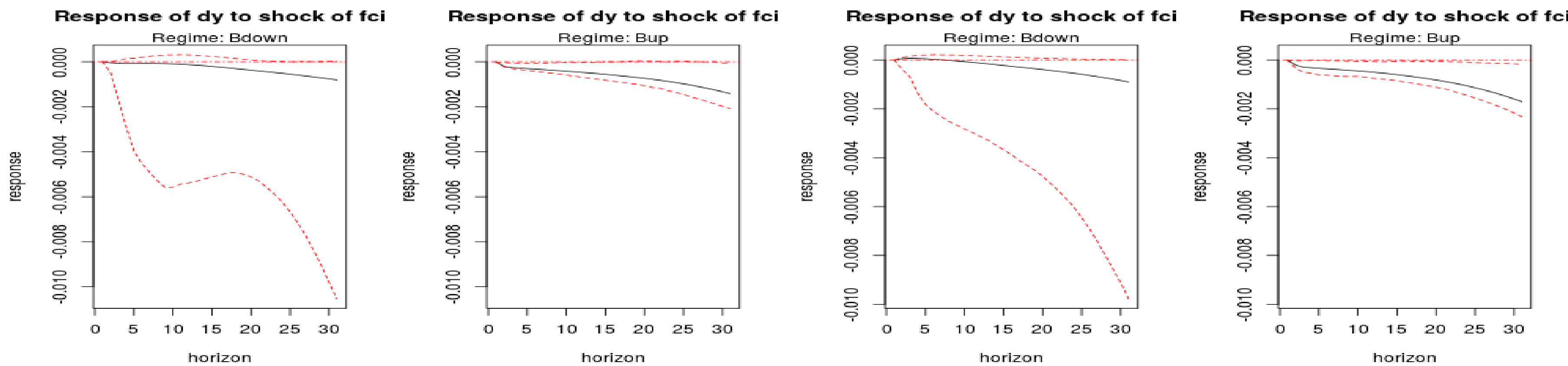


Figure 10: Spain: Response of change in output to a FCI shock of 1 s.d.; two graphs on the left: interest rate in levels with low stress regime (Regime: Bdown) and high stress regime (Regime: Bup); two graphs on the right: interest rate in first differences with low stress regime (Regime: Bdown) and high stress regime (Regime: Bup)

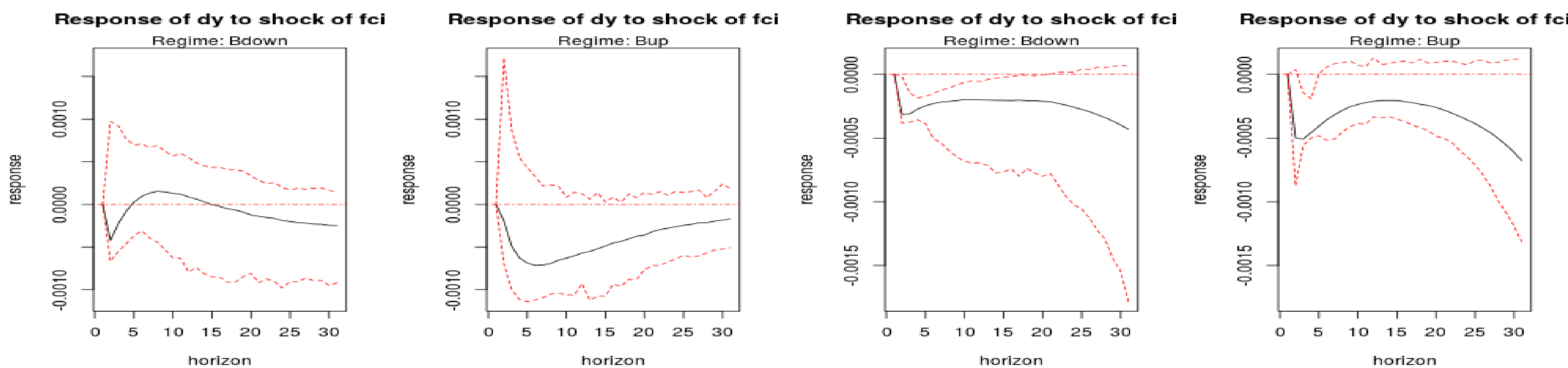


Figure 11: Italy: Response of change in output to a FCI shock of 1 s.d.; two graphs on the left: interest rate in levels with low stress regime (Regime: Bdown) and high stress regime (Regime: Bup); two graphs on the right: interest rate in first differences with low stress regime (Regime: Bdown) and high stress regime (Regime: Bup)

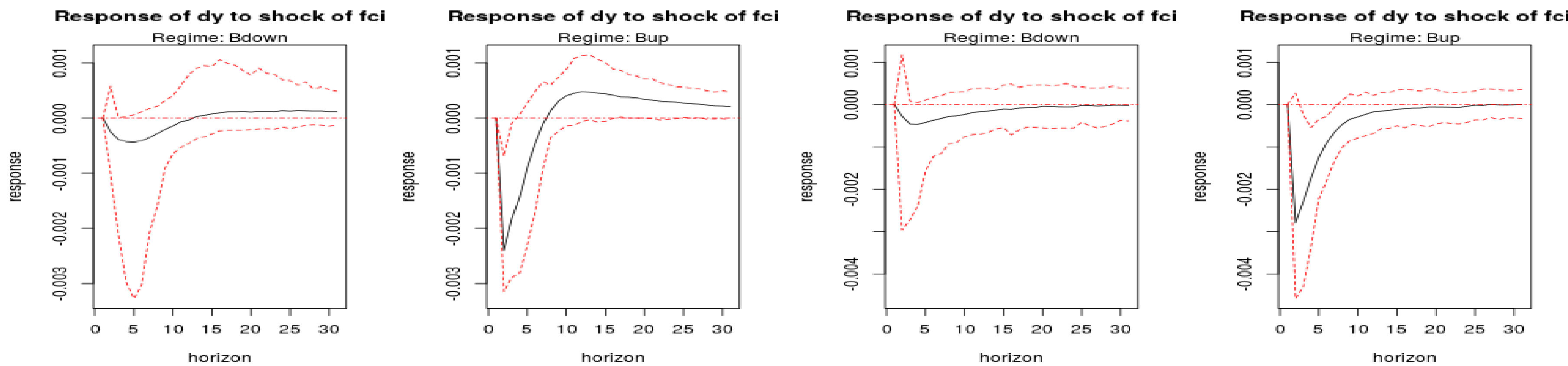


Figure 12: Germany: Response of change in output to a FCI shock of 1 s.d.; two graphs on the left: interest rate in levels with low stress regime (Regime: Bdown) and high stress regime (Regime: Bup); two graphs on the right: interest rate in first differences with low stress regime (Regime: Bdown) and high stress regime (Regime: Bup)

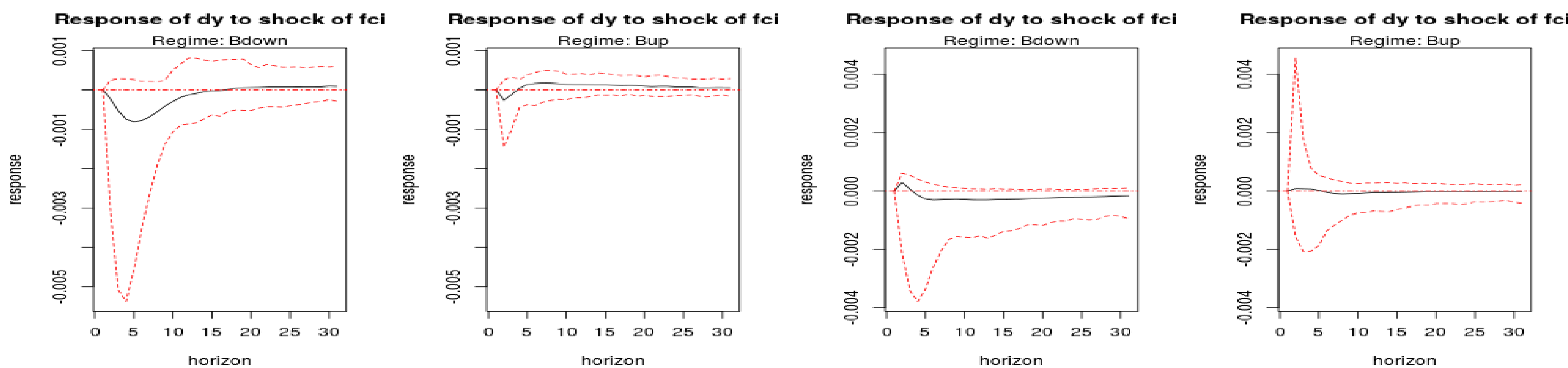


Figure 13: France: Response of change in output to a FCI shock of 1 s.d.; two graphs on the left: interest rate in levels with low stress regime (Regime: Bdown) and high stress regime (Regime: Bup); two graphs on the right: interest rate in first differences with low stress regime (Regime: Bdown) and high stress regime (Regime: Bup)

As can be seen from the figures for Spain, Italy and Germany a positive shock to the FCI index (i.e. an increase in financial stress) leads to an economic contraction as change in output becomes negative. Furthermore, reflecting non-linearities of financial market shocks, the consequences of increasing financial stress are more severe in times of financial stress compared with a period of tranquility in financial markets. This result can be observed for the three countries for the benchmark model with the interest rate in levels, as well as for the model with the interest rate in first differences.

However, while the effect is negative for Spain and Italy in the beginning, and remains negative in the long run, Germany's output growth turns positive within 10 quarters in both regimes. In the first difference model a similar effect can be observed: while growth remains negative in Italy and Spain, the model would suggest that the German economy recovers quickly, though output is also strongly affected by an FCI shock in a financial stress regime, in the earlier impact phase. This so far confirms our model predictions of sect. 4.

The effects of a FCI shock in France, on the other hand, are not what one would expect from our theoretical model and the MRVAR and IRs of the other countries. The empirical results for France would suggest that the economy would recover more quickly from FCI shocks than the other countries, and also a bit more in times of financial market stress as compared to a tranquil period. A similar result for France has been found in Mittnik and Semmler (2013). As it is argued there, the different results for France appear to come from the fact that France has a large public sector, and a stronger public policy impact on the financial sector so that FCI shocks might not show the usual features.

6 Implications for macro policies

As we empirically showed in sect. 2, using the GVAR approach, Europe currently faces the acute danger of adverse effects of debt deflation on output and employment more strongly than Japan and the US. Our study also demonstrated, though the disinflation or deflation of product prices is likely to trigger a protracted period of recession and output decline, there are, as the model variants of sects. 4.1 and 4.2 demonstrated, additional financial risk drivers and a possible change into a high financial stress regime, with high credit costs and low credit flows, that create the danger of a protracted period of output and employment decline. This switch into a new regime is likely to be triggered and amplified by overleveraging and actual defaults, insolvencies of firms and banks, loan losses and fire sales of assets, leading to rapid credit, output and employment contractions.[63] We have also shown that debt stabilization might become more difficult with those deflationary and financial risk drivers.

As to the debt deflation spiral itself, disinflation and deflation has been built into the model variant of sect. 3, replicating roughly the empirical results of the NK literature, though in a more short cut way, with modeling the rate of change of the inflation rate. We could replicate the fact for some countries that in the Great Recession and the aftermath prices did decline less than what had been expected given the large output and employment slacks. Our empirics of sect. 5, estimating non-linearities and asymmetries of effects of shocks in a MRVAR has also underlined the perils of debt deflation when it is coupled with an addition financial market stress in a regime of credit spread rising and credit flows reduced, entailing an output and employment decline.

When using our four dimensional MRVAR model with the financial stress as transition variable we showed empirically that there is a high risk that the vulnerability of the financial sector and financial stress may trigger a regime change in the financial market - output link. Though short term interest rates can be kept down, following the Taylor rule for the central bank, real interest rate might rise due to deflation and financial stress and credit spreads could nevertheless shoot up, triggering a transition to a financial crisis regime in Euro-area countries.[64] What appears to

[63] For a similar conclusion on extensive historical study of deflationary period, Borio et al (2014) come to a similar conclusion.

[64] Many earlier NK studies often allow the interest rate to to fall through the central bank's monetary policy rule. A new type of NK literature seems to emerge that take into account the role of financial market risk for risk premia and credit spread, see Furlanetto et al (2014).

be important in this context is the degree of the financial sector's leveraging and overleveraging, see appendix 3. In fact the latter issue, as our sect. 4 shows, the rise of vulnerability of the financial sector, the decline of credit flows, and an increase of financial market stress, are important amplifying forces to also trigger adverse macro feedback loops. This appears to become then a more severe accelerating force interacting with the slowly moving debt deflation process.

In this context we can spell out some implications of our modeling results for macro policies. Pursuing sovereign debt consolidation policies might work under the conditions corresponding to a normal path of the economy, such as sketched in appendix 2, and it also might temporally work under the condition shown in the right trajectory in figure 8. Yet, in a regime of high financial market stress, deflation and rising real interest rate, and a larger jump in the risk premia (and discount rates), with capital gains and net worth falling, the banking system under financial stress, and with central banks constrained to undertake an unconventional intervention into asset markets, the strong macro feedback loops create great challenges for debt fiscal consolidation policies, and they are likely not to be successful, under the condition of the left trajectory of figure 8.[65] In this case, output and employment multipliers are strong and are likely to trigger adverse and amplifying feedback loops.

Yet, as appendix 2 might demonstrated, a declining credit risk and credit spreads, possibly engineered by monetary policy of quantitative easing, can reduce adverse macro feedback effects and support policies of debt sustainability output and employment growth. As recent empirical literature on EU periphery countries have shown, reducing the risk premia and credit spread appears to be an important escape route from high financial stress and default risk. Thus, monetary policy of quantitative easing, which was pursued in the US in the aftermath of after the Great Recession, and introduced in Euro area countries with the ECB policy starting in January 2015, seems to be very important to escape from a deflationary trends and high financial stress-high credit spread regimes.

What might be needed, however, is as Brunnermeier and Sannikov (2014b) argue, a more selective monetary policy, as well as selective asset market and credit market policies to overcome regional and local bottle necks in lending and borrowing. We have shown that output and employment gaps, arising for example in financially caused recessionary periods, give rise, together with low vacancy rates, to low job finding rates and high unemployment. The decentralized matching mechanisms of the labor and credit markets in the Euro-Area economies may result in quite different success of aggregate policies. More specific growth and labor market policies may be appropriate. This seems to be in contrast to the observations of the policy effects in the long lasting period of output and employment decline in the US after the Great Recession, where aggregate macro policies, because of the more fluid financial, credit and labor market matching mechanisms appear to have worked better.

7 Conclusions

As shown, the EU-area countries, with large debt overhang, seems to have entered, after the period of output and employment decline during the great recession 2007-9, a new period of disinflation, and even deflation. As shown disinflation and deflation leads to a rising real interest rate, exacerbating the leveraging problem. There appeared to be a risk of a regime shift into high financial stress and rising credit spreads, possibly accompanied by strong adverse macroeconomic feedback loops. To investigate the consequences of overleveraging and the potential for destabilizing effects from deflation and financial – and real–sector interactions we introduce first a theoretical model, and demonstrate, with the presence of regime-dependent macro feedback relations, the possibility of an unstable dynamics and downward spirals. In order to capture the heterogeneity in the Euro-area, we introduce decentralized matching mechanisms on the labor and financial markets. We might then conjecture that those dynamics are different for the Northern core countries in contrast to periphery countries.

[65] For details of such scenario of a failing debt stabilization in some EU countries, see Semmler and Semmler (2013).

If we admit debt overhang and disinflation and deflation, and endogenously generated credit constraints, risk premia and credit spreads, we can observe in the theoretical model destabilizing effects, with persistent contractions and no debt sustainability. Moreover, with the rise of credit and financial stress, large contractions, with protracted periods of large and persistent unemployment spells, an unsustainable debt dynamics can arise. This is likely to be exacerbated when there are significant adverse macro economic feedback effects of credit spreads and financial stress on employment, consumption, investment demand, and utilization of capacity. On the other hand, we showed that with increasing rates of inflation, low financial stress and low interest rates are conducive to deleveraging, debt sustainability and higher growth of output and employment.

Overall, as we showed, in particular southern countries need higher inflation and growth rates to overcome the debt deflation and financial stress problems. Differences in the core and periphery countries need to be taken into account in particular when employment and credit policies are designed. Given the very heterogeneous matching mechanisms on the labor and credit markets, more selective policies seem to be needed.

Empirically, deflationary trends in the Euro-Area were explored with MRVAR models with regime dependent IRs to study time series effects of risk drivers and investigated nonlinear relationships in higher dimensional regime change models where a measure of financial stress was the regime defining variable. In general, MRVARs,[66] help us understand what happens in different regimes and one can observe state dependence of fiscal and monetary policy effects: one can predict a quite different impact of policies in contractions, in particular on output, employment and inflation rates as well as financial stress, as compared to expansions.

Some regime dependence of macro laws are studied in appendix 5. As we demonstrate there, using a low dimensional VSTAR method, the credit-output link, the Phillips-curve and Okun's Law are quite regime dependent and need to be looked at in a regime dependent context. As to the credit-output link, negative output or credit shocks trigger much larger negative responses to shocks in a high leveraging regime with excessive debt, this holds for both when GDP or credit are shocked. Similar differences can be seen for the Phillips curve and Okun's law in a period of high financial stress as compared to a low stress regime. We also show that significant differences of those macro laws can be uncovered for Southern as compared to Northern EU-area countries.

Methodological, to explore the linkages between output, inflation, labor and financial markets, we employ a new numerical solution method, called non-linear model predictive control (NMPC), to solve our model variants. We have used this new numerical procedure in macro economics that helps to solve model variants with constraints and regime changes in finite horizon decision models. NMPC allows one to solve those model variants by providing global solutions to higher dimensional models studied and it is less constrained by the curse of dimension.

[66] For further results using MRVARs, see Mittnik and Semmler (2012a,b, 2013), for results on VSTARs, see Schleer and Semmler (2013), and Schleer et al. (2014).

Appendix:

1. Numerical Procedure

For the numerical solution of the optimal control problem we do not apply here the dynamic programming (DP) approach as in Ernst and Semmler (2010) or DYNARE as used to solve DSGE model. Though DP method has the advantage that a global solution to the optimal control problem can be found, by first computing an approximation to the optimal value V and then the optimal control, and its time path, is computed from V. For a detailed description of the specifics of the DP algorithm we are using we refer to Gruene and Semmler (2004). The main disadvantage of DP, however, is that its numerical effort typically grows exponentially with the dimension of the state variable. Hence, even for moderate state dimensions it may be impossible to compute a solution with reasonable accuracy.[67]

A remedy to this problem can be obtained by using nonlinear model predictive control (NMPC). Instead of computing the optimal value function for all possible initial states, NMPC only computes single (approximate) optimal trajectories. In order to describe the method, let us abstractly write the optimal decision problem as

$$\text{maximize} \quad \int_0^\infty e^{-\rho t}\ell(x(t), u(t))dt,$$

where $x(t)$ satisfies $\dot{x}(t) = f(x(t), u(t))$, $x(0) = x_0$ and the maximization takes place over a set of admissible control functions. By discretizing this problem in time, we obtain an approximate discrete time problem of the form

$$\text{maximize} \quad \sum_{i=0}^{\infty} \beta^i \ell(x_i, u_i), \tag{21}$$

where the maximization is now performed over a sequence u_i of control values and the sequence x_i satisfies $x_{i+1} = \Phi(h, x_i, u_i)$, Here $h > 0$ is the discretization time step, $\beta = e^{-\rho h}$ and Φ is a numerical scheme approximating the solution of $\dot{x}(t) = f(x(t), u(t))$ on the time interval $[ih, (i+1)h]$. For details and references in which the error of this discretization is analyzed we refer to Gruene and Semmler (2004).

The idea of NMPC now lies in replacing the maximization of the infinite horizon functional (1) by the iterative maximization of finite horizon functionals

$$\text{maximize} \quad \sum_{k=0}^{N} \beta^i \ell(x_{k,i}, u_{k,i}), \tag{22}$$

for a truncated finite horizon $N \in \mathbb{N}$ with $x_{k+1,i} = \Phi(h, x_{k,i}, u_{k,i})$ and the index i indicates the number of the iteration, cf. the algorithm below. Note that neither β nor ℓ nor Φ changes when passing from (1) to (2), only the optimization horizon is truncated.

Problems of type (2) can be efficiently solved numerically by converting them into a static nonlinear program and solving them by efficient NLP solvers, see. Gruene and Pannek (2012). In our simulations, we have used a discounted variant of the MATLAB routine `nmpc.m` available from `www.nmpc-book.com`, which uses MATLAB's `fmincon` NLP solver in order to solve the resulting static optimization problem.

Given an initial value x_0, an approximate solution of (1) can now be obtained by iteratively solving

(2) as follows:

[67] Another global algorithm that works with gridding and computation of the value function and computation of the decision variables at each grid point, is used in Gavin et al (2013), where a New Keynesian model is solved globally. They point out quite different solutions far from the steady state as compared to solutions close to the steady state. Thus, they also show that nonlinearities matter. Yet for their algorithm it also holds that there is a curse of dimension.

(1) for i=1,2,3,...
(2) solve (2) with initial value $x_{0,i} := x_i$ and denote the resulting optimal control sequence by $u^*_{k,i}$
(3) set $u_i := u^*_{0,i}$ and $x_{i+1} := \Phi(h, x_i, u_i)$
(4) end of for-loop

This algorithm yields an infinite trajectory x_i, $i = 1, 2, 3, \ldots$ whose control sequence u_i consists of all the first elements $u^*_{0,i}$ of the optimal control sequences for the finite horizon subproblems (2).

Under appropriate assumptions on the problem, it can be shown that the solution (x_i, u_i) (which depends on the choice of N in (2) converges to the optimal solution of (1) as $N \to \infty$. The main requirement in these assumptions is the existence of an optimal equilibrium for the infinite horizon problem (1). If this equilibrium is known, it can be used as an additional constraint in (2), in order to improve the convergence properties.

However, recent results have shown that without a priory knowledge of this equilibrium this convergence can also be ensured, see Gruene (2012), and this is the approach we use in the computations in this paper. It should be noted that the references just cited treat averaged instead of discounted infinite horizon problems. However, the main proofs carry over to the discounted case, see Gruene et al. (2014). In any case, the solution generated by NMPC will always provide a lower bound for the true optimal solution. The procedure also allows for irregular impacts on the dynamics of the state variables and regime switches.[68]

2. The basic model – normal regime

The decentralized matching mechanisms on the labor market is as proposed in Merz (1995) and used in Ernst and Semmler (2010). For the matching mechanisms on the credit market we assume that there is a stream of financial funds, $\mathcal{F}_t$, determining the supply of available funds which come from domestic savings and capital inflow. The demand for funds come from households for consumption so that consumption can be smoothed inter-temporally and there is an intermediation for funds for investment. Presuming that the households obtain funding without constraints, is an assumption in order to reduce the dimension of the system, but we have still three state variables. All variables are here in real terms.

Overall, the dynamic decision problem has three decision variables and it is subject to three dynamic constraints, one for the change in employment, a second for capital accumulation, and a third for debt accumulation:

$$\max_{C_t, \mathcal{I}_t, \mathcal{V}_t} \int_0^N U(C_t, N_t)\, e^{-\rho t} = \int_0^N \left\{ \frac{C_t^{1-\eta}}{1-\eta} - e N_t^{\chi} \right\} e^{-\rho t}$$

s.t.

$$\dot{N}_t = m^L(s_t \cdot \mathcal{U}_t, \mathcal{V}_t) - \sigma N_t \tag{23}$$
$$\dot{K}_t = m^B(\mathcal{I}_t, \mathcal{B}_t) - \delta K_t \tag{24}$$
$$\dot{D}_t = r D_t - \upsilon[Y_t(K_t, AN_t) - \mathcal{F}_t - \Phi(s_t)(1 - N_t) - \zeta \cdot \mathcal{V}_t - \kappa \cdot \mathcal{B}_t] \tag{25}$$

The preferences are over consumption flows, C, and employment, N. The dynamics of eq. (23) represents the evolution of employment which is normalized to one. Equation (24) denotes the evolution of the capital stock and equation (25) represents the dynamics of aggregate debt (households and firms).[69] Our debt dynamics is written here in a way which is standard if one allows for borrowing of the private (or public sector), possibly also from abroad.[70] There could be a stochastic shock occurring along the path, for example represented by eq. (23) or eq. (24). Yet, we will neglect such shocks, except when it will lead to a regime change in the dynamics.

[68]Note that in DSGE models regime switches are also perceived as something likely to occur which some literature starts to explore now, see Farmer et al. (2008).

[69]We could also allow for sovereign debt here, though we do not specify what fraction of external debt is driven by households, firms or the public sector.

[70]see Blanchard and Fischer (1989, ch. 2) and Blanchard (1986).

The terms represent C_t : aggregate consumption, Y_t: aggregate production, A : (exogenous) labor productivity, $\mathcal{F}_t$: available financial funds through savings and external borrowing, $\mathcal{B}_t = \mathcal{F}_t - C_t$: offered bonds to firms, D_t : the stock of external debt, r : the (exogenous given) interest rate,[71] N_t: employment and $\mathcal{V}_t$: vacancies. In addition to costly search on the labor market, issuing bonds adds another cost factor to the macroeconomic resource constraint, with per-period flow costs for unmatched bonds measured by κ. Whereas D_t represents external debt, the term [.] is external borrowing (> 0) or repayment (< 0), in the former case used for excess spending over domestic income, and moreover $\mathcal{F}_t = \mu C_t$, $\mu > 1$. Thus, consumption can be smoothed inter-temporally, but investment funds might be restricted. This means investment is more scrutinized through decentralized financial market matching mechanisms,[72] but if there is a consumption boom, more investment funds will also be available.[73]

Moreover, $\Phi(s)$ is taken fixed, equal to $s = 0.2$. The function $m^L(s_t \cdot \mathcal{U}_t, \mathcal{V}_t)$ in eq. (1) is a a decentralized matching function on the labor market. Given the decentralized S&M outcome by $m^L(s_t \cdot \mathcal{U}_t, \mathcal{V}_t)$ the job finding rate of the unemployed will be $m^L(s_t \cdot \mathcal{U}_t, \mathcal{V}_t)/\mathcal{U}_t$ which will depend, fixing the search intensity, on the vacancies posted by firms and the unemployment rate.[74] The job finding rate is thus the ratio of the numbers of new hires divided by the number of workers searching for jobs. With higher unemployment and lower vacancies the job finding rate is lower.[75]

On the credit market there is also a decentralized matching mechanism defined by $m^B(\mathcal{I}_t, \mathcal{B}_t)$, which represents the decentralized matching mechanisms, as S&M function, for the credit market. Both, the S&M function for the labor and for the credit markets are Cobb-Douglas, with exponents q_0=0.5 and $q_1 = 0.5$. The parameters σ, δ are the separation rate and depreciation rate of capital, and υ is our regime switching parameter which will be either 1 or 0, depending on the degree of leverage the economy reveals.

As mentioned, in order to avoid a fourth decision variable, we have made the external supply of investment funds for firms a function of the total supply of funds. Given then the external funding and the consumption decision, the remaining funding can be used for providing bond offering to be matched with the bond demand arising from firms‘ desired investment $\mathcal{I}_t$. Funding for consumption will be available from domestic and external sources, but investment funding will be obtained on the credit market by the decentralized matching process on the credit market. Note that in this first step we do not have constraints on consumption smoothing.

Thus in this basic model here, we assume that consumption is a direct choice variable and investment is expressed as intended investment, $\mathcal{I}$, to be matched with the supply of bonds, the supply of funds for bonds given by $F_t - C_t$. As mentioned, this might be a reasonable assumption that allows us to work with a lower dimensional system. It also means that the screening and monitoring of investment funding takes place more extensively than funding for optimal consumption. In this context here, consumption is only indirectly constrained, namely through the generated output, its increase is given through the accumulation of capital stock through investment.

We can solve for the basic model using our NMPC procedures. For the basic model, representing a normal situation, where we have no regime switching, the two previous scenarios of sect. 4.1 and 4.2 do not necessarily prevail if strong monetary policy action are undertaken to reduce the interest rate and the financial market stress. This could occur for example through a policy of quantitative easing, which the US Fed has exercised and on which the ECB has embarked on since the beginning of the year 2015.

For illustrating the potential effects of such a policy we presume that the actual interest rate can be brought down to 3 percent and is kept there by the central banks monetary policy actions. As parameters for the NMPC solution we assume again: $\mu = 1.3$ and $\beta = 035$, $\kappa = 0.1$, $\rho = r = 0.03$, $\delta = 0.03$, $\sigma = 0.04$, $\alpha = 0.36$, $A = 1$, $\xi = 0.07$, $\chi = 5$, $e = 1$. The parameter υ is set to one.[76]

[71] Note that the interest rate could be derived from a monetary policy rule, such as the Taylor rule, as for example in Gavin et al (2013). In the first step here we do not consider price dynamics.

[72] This in principle allows us to study more properly the heterogeneity of the EU-area credit market.

[73] This for example, was likely to be the situation in Spain before the financial meltdown of the years 2007-9. Of course, there are likely to be constraints for households' borrowing as well.

[74] This gives rise to the usual Beveridge curve

[75] For details, including also time varying labor market participation rates, see Christiano et al (2014).

[76] The parameter υ can be used as switching parameter, and $\upsilon = 0$ would indicate the binding of credit constraints.

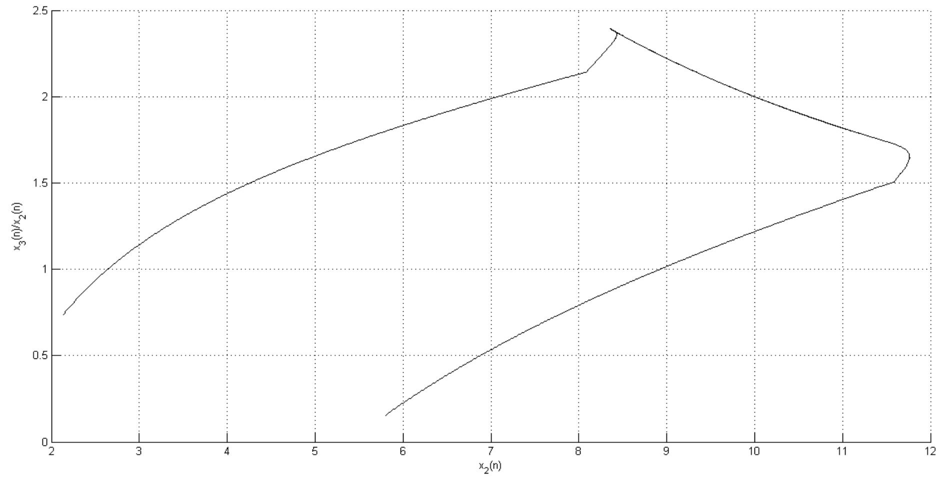


Figure 14: Dynamic solutions, horizontal axis is capital stock, vertical the debt to capital stock ratio, for $\upsilon = 1$, global solution for two initial conditions, convergence to some steady state (where the trajectories merge)

Applying the NMPC procedure, gives us a solution such as depicted in figure 14. In figure 14 the solution of eqs. (23)-(25) is shown which again represents an approximation of an infinite horizon model by a finite horizon model, using a time horizon $N = 6$. As shown in figure 14 there is a steady state at about $K = 8.5^*$ and $(D/K)^* = 2.25$. The steady state is unique and all initial conditions for the state variables would converge toward that point.

As shown in the figure, debt could be stabilized and a steady state debt to capital ratio could be reached, see the upper converging point of the trajectories. We have chosen here parameter values which give a large steady state leveraging. Yet, we hereby have assumed that we have a regime of low interest rates which is supposed to be kept there at the low level.[77] The yield on debt is $r = 0.03$, equal to the discount rate, and the interest rate stays low even if leveraging is increasing and becomes larger. This success could presumably be an outcome of strong and persistent monetary policy actions.

3. GIRF algorithm

We follow the approach of Caggiano (2015) in computing the GRIF. The algorithm has also been outlined in Semmler and Haider (2015).

1. Take the set of all observations which allows us to build $T - p + 1$ histories to draw from (with replacement). The histories are split into M regime-subsets $(\Omega_1, \ldots, \Omega_M)$ according to the regime they belong to.

2. Take a set of histories (Ω_i) out of one of the M subsets from step (1) and compute the regime-dependent Variance-Covariance Matrix Σ_i.

3. Cholesky decompose Σ_i which gives $\Sigma_i = C_i C_i'$ and orthogonalize the regime-dependent residuals to get the structural shocks: $e_i = C_i^{-1} \epsilon_i$

4. Draw a history $\omega_j \in \Omega_i$.

This case is further explored in a companion paper, we will stay here with $\upsilon = 1$.

[77] The interest rate could be close to the zero bound, which would make the debt sustainability and convergence even more likely.

5. From e_i draw a set of n four-dimensional structural errors $e_i^* = (e_{it}, \ldots, e_{it+n})$ with replacement, where the contemporaneous correlation of the structural errors is taken into account. Afterwards transform the residuals back into their reduced form representation: $\epsilon_i^* = C_i e_i^*$.

6. Use the history from step (4) and the structural errors from step (5) to simulate the model with the parameters from the MRVAR model.

7. Take the structural errors from step (5) and add an additional shock in period t : $e_i^v = (e_{it} + v_t, \ldots, e_{it+n})$. Then compute the reduced form errors as in (5).

8. Use the history from step (4) and the structural errors from step (7) to simulate the model.

9. Repeat steps (5) through (8) $R = 100$ times and take the average of the simulations from step (6) and from step (8). The difference of the averages represents the GIRF for history j.

10. Repeat steps (2) through (9) $l = 300$ times for regime i where the histories are drawn from Ω_i with replacement. Take the average over all estimated $GIRF^i$ $(GIRF^{i,1}, \ldots, GIRF^{i,l})$ which represents the GIRF for regime i.

11. Repeat steps (2) through (10) for all regimes to get the GIRF for all M regimes.

12. The confidence intervals are computed by taking the 5% and 95% percentile of the densities of the simulated GIRF $(GIRF^1, \ldots, GIRF^l)$ for each regime.

4. Measuring excess leveraging

Since one issue of the debt deflation is the build-up of excess debt, we here briefly want to sketch how this can be measured. Excess debt is defined as difference of actual debt and sustainable debt.[78]

Sustainable Debt

Stein (2012) shows how the optimal debt ratio can be derived in the simplified case of logarithmic utility. The stochastic differential equation for net worth is (A1).

$$dX(t) = X(t)[(1 + f(t))(dP(t)/P(t) + \beta(t)dt) - i(t)f(t) - cdt] \qquad (A1)$$

$X(t)$ =Net worth, $f(t)$ = debt/net worth = $L(t)/X(t)$, $dP(t)/P(t)$ = capital gain or loss, as stochastic, $i(t)$ = interest rate, also stochastic, $(1 + f(t))$ = assets/net worth, $\beta(t)$ = productivity of capital. Hereby $c(t) = C(t)/X(t)$, consumption/net worth, c is taken as fixed. Let the price evolve stochastically as

$$dP(t) = P(t)(a(t)dt + \sigma_p dw_p) \qquad (A2)$$

where drift a(t) will depend upon the Model I or II. The interest rate also evolves as stochastic process

$$i(t) = idt + \sigma_i dw_i \qquad (A3)$$

substitute (A2) and (A3) in (A1) and derive (A4)

$$dX(t) = X(t)[(1 + f(t))(a(t)dt + \beta(t)dt) - (if(t)dt + c\,dt)] + [(1 + f(t))\sigma_p dw_{p-} \sigma_i f(t)dw_i]$$

$$dX(t) = X(t)M((t)dt + X(t)B(f(t)) \qquad (A4)$$

$$M(f(t)) = [(1 + f(t)(a(t)dt + \beta(t))dt - (if(t) + c)]$$

[78] For details, see Schleer et al. (2014).

$$B(t) = [(1 + f(t))\sigma_p dw_p - \sigma_i f(t) dw_i]$$

$$B^2((t) = (1 + f(t)^2\sigma_p^2 dt + f(t)^2\sigma_i^2 dt - 2f(t)(1 + f(t))\sigma_i\sigma_p dw_p dw_i$$

$$Risk = R(f(t)) = (\frac{1}{2})[(1 + f(t))^2\sigma_b^2 + f(t)^2\sigma_i^2 - 2(t)(1 + f(t))\sigma_b\sigma_i\rho)]$$

$M(f(t))$ contains the deterministic terms and $B(f(t))$ contains the stochastic terms. To solve for $X(t)$ consider the change in $lnX(t)$ (A5). This is based upon the Ito equation of the stochastic calculus. As Stein shows using the logarithmic criterion one does not need to use dynamic programming. The expectation of $d\,lnX(t)$ is (A6).

$$dlnX(t) = (1/X(t))dX(t) - (1/2X(t)^2)(dX(t))^2 \qquad (A5)$$

$$E[d(lnX(t))] = [M((t)] - R[((t)]dt] \qquad (A6)$$

Equ. (A6) represents a mean-variance formulation. The correlation $\rho\,dt = E(dw_p dw_i)$ is negative, which increase risk. $(dt)^2 = 0$, $dwdt = 0$.

The optimal debt ratio f^* maximizes the difference between the Mean and Risk.

$$f^* = argmax[M(f(t)) - R(f(t))] = [a(t) + \beta(t) - i] - [(\sigma_p^2 - \rho\sigma_i\sigma_b)]/\sigma^2 \qquad (A7)$$

$$\sigma^2 = \sigma_i^2 + \sigma_p^2 - 2\rho\sigma_i\sigma_p$$

On model version of Stein (2012) assumes mean reversion so that the price $P(t)$ has a trend rt and a deviation $y(t)$ from it (A8). The deviation $y(t)$ follows an Ornstein-Uhlenbeck ergodic mean reverting process (A9). Coefficient α is positive and finite. The interest rate is the same as in model II.

$$P(t) = P(0)exp[rt + y(t)] \qquad (A8)$$

$$dy(t) = -\alpha y(t) + \sigma_p dw_p \qquad (A9)$$

Therefore using the stochastic calculus $a(t)$ in Model I is the first term in (A10)

$$dP(t)/P(t) = (r - \alpha y(t) + (\frac{1}{2}\sigma_p^2)dt + \sigma_p dw_p \qquad (A10)$$

Substitute (A10) in (A7) and derive (A11), the optimal debt ratio in Model I.

$$f^*(t) = [(r - i) + \beta - \alpha y(t) - (\frac{1}{2})\sigma_p^2 + \rho\sigma_i\sigma_p]/\sigma^2 \qquad (A11)$$

Hereby Stein considers $\beta(t)$ as deterministic.

Another Model on capital gains and returns presumes the price equation is (A12). The drift is $a(t)dt = \pi\,dt$ with a diffusion term $\sigma_p dw_p$.

$$dP(t)/P(t) = \pi dt + \sigma_p dw_p \qquad (A12)$$

The optimal debt ratio $f^*(t)$ is (A13). Again, consider $\beta(t)$ as deterministic.

$$f^*(t) = [(\pi + \beta(t) - i) - (\sigma_p^2 - \rho\sigma_i\sigma_p)]/\sigma^2 \qquad (A13)$$

$$\sigma^2 = \sigma_i^2 + \sigma_p^2 - 2\rho\sigma_i\sigma_p$$

Empirical measures of excess leveraging of banks can be obtained as discussed in Schleer et al (2014). There it is shown what empirical variables are needed to compute sustainable debt and actual debt, both as normalized debt ratios.

Measuring issues

$$((r-i)+\beta-\tilde{\beta}+0.5\sigma_r^2+\rho*\sigma_i*\sigma_r)/\sigma^2 \tag{26}$$

with r=capital gains , i= long-term government bond yield, β= productivity of capital, $\tilde{\beta}$ = demeaned beta (beta deviation), σ_r^2 = Half of the square of the demeaned capital gain, $\bar{i}$= demeaned interest rate, $\bar{r}$= demeaned capital gain, ρ= correlation of i and r, σ_i= standard deviation of interest rate, σ_r= standard deviation of capital gain, σ= risk element =$(\sigma_i+\sigma_r-2(\rho\bar{i}\bar{r}))$.

Capital gains, denoted by the variable r, are calculated as the percentage change in the stock market capitalization (market cap) of the bank during the period. The market cap data is given quarterly and computed as the product of the stock market price and the common shares outstanding. The common shares outstanding is the difference between issued shares and treasury shares. The market cap is subject to stock market swings. To eliminate these, and to obtain the trend of capital gains, the Hodrick-Prescott-Filter with parameter $\lambda = 1600$ matched to the quarterly frequency of the data is applied. The filtered capital gain is then given by the percentage change between the year-end market caps.

The long-term government bond yield i corresponds to the long-term (9/10-year) treasury yield of the country the bank is mainly situated at and is given in percent. The productivity of capital β is calculated by dividing the bank's net income from the annual shareholders' equity for each period. More precisely, we consider the net income after preferred dividends which the bank uses to calculate its basic earnings per share. The annual shareholders' equity is thereby given as the sum of preferred stock and common shareholders equity. Once again year-end data is used. To calculate the beta deviation $\tilde{\beta}$, the difference between each period's beta from the mean beta over all periods is computed. The demeaned interest rate $\bar{i}$ and demeaned capital gain $\bar{r}$ are calculated identically.

The correlation ρ of the capital gains r and the interest rate i is calculated over the entire period and then used as a constant value over the periods. Similarly, the standard deviations σ_i and σ_r of the interest rate and capital gain, are also constant over the periods. The risk element σ of the formula is given as the sum of the standard deviations σ_i and σ_r deducting twice the value of the variances of the interest rate, capital gain and the correlation between them.

The actual debt ratio is calculated as the average yearly long term debt balances divided by the average yearly total assets. When both the actual and sustainable are normalized, one can take the difference of the two to obtain excess leveraging, which plays an important role in our estimation of the vulnerability of the economic sectors considered.

5. Regime dependent macro laws

Using two or three dimensional VARs and IRs we can estimate a small-scale macro-econometric models with multiple regimes, using vector smooth-transition auto-regressive techniques (VSTAR) and applying it to data for the US and EU country groups. But note, though the smooth transition auto-regression method (STAR) model, is able to estimate regimes and to capture different dynamic properties over time and across regimes, so far only low dimensional macro problems have been addressed. In general, regimes can be estimated through an indicator function, a Markov switching model or a STAR model (a smooth transition regression model). A specific regime can be defined by an output regime (low and high output growth rates),[79] financial market regime (low and high financial market stress),[80] or with respect to low and high leveraging (see Schleer et al. 2014). We here refer to a VSTAR model, as originating in Teräsvirta and Yang (2013) and used in Schleer and Semmler (2013). There are usually only two variables, where one variable is (an) endogenous transition variable. Thus the regime change variable can be an endogenous variable, but if it is an exogenous variable, one can employ three variables. In the estimations below, we model a special case where one transition function governs the whole VSTAR system, with an endogenous or exogenous transition variable. The transition function captures the non-linearity of the transition

[79] Mittnik and Semmler (2012)
[80] Mittnik and Semmler (2013) and Schleer and Semmler (2013)

variable with respect to the other variables of the system and, hence, looks as follows:

$$\mathbf{g}(fs_t|\gamma, c) = [1 + \exp(-\gamma(fs_t - c))]^{-1}, \ \gamma > 0 \tag{27}$$

which is bounded between zero and one, is monotonically increasing in fs_t, depends on the transition speed (γ), the location parameter (c) and the transition variable (fs_t). The transition variable is either a contemporaneous or lagged variable, and can be an endogenous or exogenous variable. The parameter γ as well as c are estimated. The location parameter c defines the threshold. We usually evaluate two regimes, $m = 2$, with :

$$TV_{t-d} < c \text{ (regime below } c)$$
$$TV_{t-d} \geq c \text{ (regime above } c)$$

As time series data for financial stress one can use the IMF FSI or the ZEW FCI. The latter has extensively included banking variables. The time series for the IMF FSI we are using below covers the period 1980 to 2012. The ZEW Financial Condition Indix (FCI) for the Euro- area reflects better financial sector conditions and stress. More than the FSI, the FCI focuses on the banking sector. This newly compiled data set relies on 21 financial sector series for each country. This index also tracks market volumes, particularly within the banking sector, as well as prices. For instance, the FCI includes the annual growth rate of assets over liabilities, which represents available bank collateral; the ratio of short- over long-term debt securities issued by banks; and the annual growth rate of bank lending to the private sector as well as a divers set of interest rates. Such an inclusion of banking-related factors with a strong link to the economic downturn improves the accuracy of our indices. Most of the variables are available at the country level, some are Euro-area aggregates. To account for a fairly high correlation across some variables, the ZEW FCI is established using a dynamic factor model to extract the common factor (for each country).[81] Next, let us explore some non-linear macro relationships

5.1 Regime dependent credit - output link

As figure 15 demonstrates, for the euro area as a whole, the impulse-responses to both negative GDP and negative credit shocks differ significantly between the two regimes. As regime defining variable we have used the time series data on excess leveraging provided by Schleer et al (2014) of the EU banking system, see appendix 4 for the measures. We define two regimes, one regime with no excess debt and another with excess debt (overleveraging) of the EU Countries' banking system. Then we study the impact of GDP shocks on credit flows in those two regimes, using regime dependent VARS and IRs.

As can in general be seen in figure 15, negative output or credit shocks trigger much larger effects in the high leveraging regime with excessive debt, both for GDP and for credit adjustments. Similarly, credit adjusts downwards much faster in response to a negative GDP shock when the Euro area is in conditions of high leveraging regime.

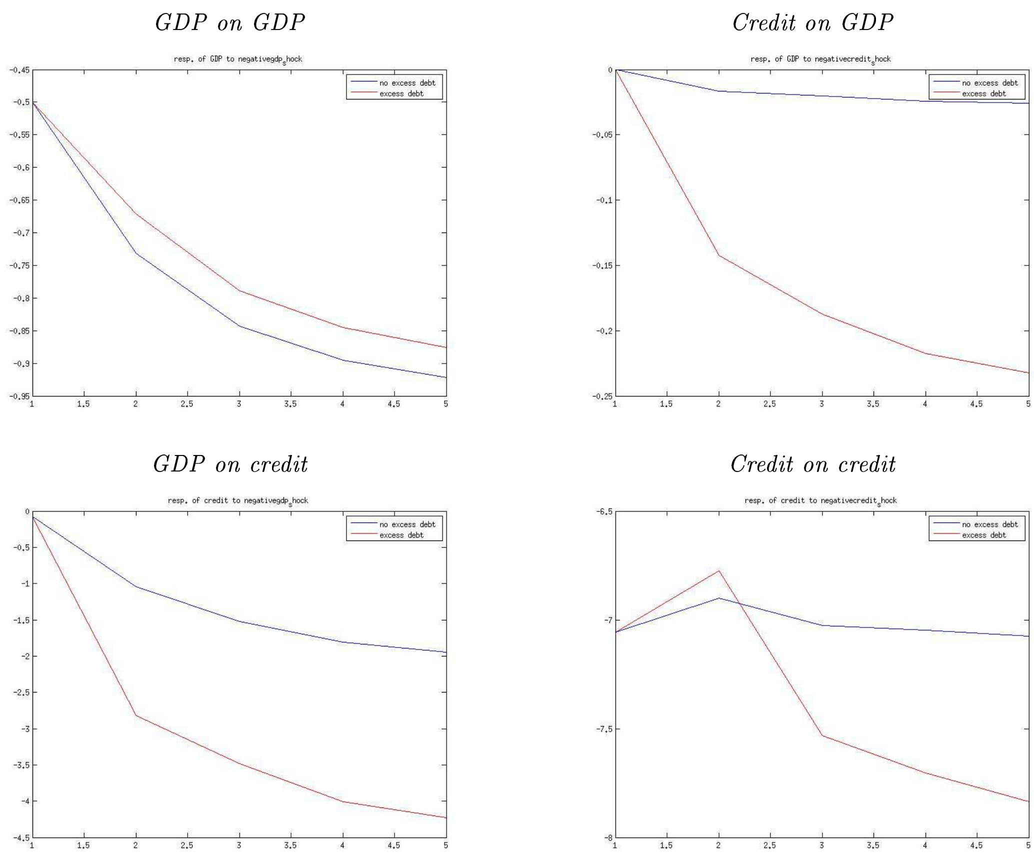
Similar results, not reported here, are obtained when looking specifically at peripheral (Southern) Euro-area countries. Here again, adjustments to GDP and credit are much stronger in a high-financial stress regime than under normal borrowing conditions, leading to significant responses of both GDP and credit to a negative credit shock.

5.2. Regime dependent Phillips Curve

We carry out the same exercise for the Phillips curve, for the Euro-area, also distinguishing between core and periphery countries. Again, the Euro-area periphery group aggregates information for Greece, Italy, Portugal, Spain and Ireland whereas the Euro-area core group sums up unemployment and inflation data for Austria, France and Germany.[82]

[81] For a detailed description of sources and transformations of the data and variables, see Schleer and Semmler (2013).

[82] Note that for the subsequent exercise we take the ZEW FCI, since the IMF FSI does not cover sufficiently the EU countries of interest.

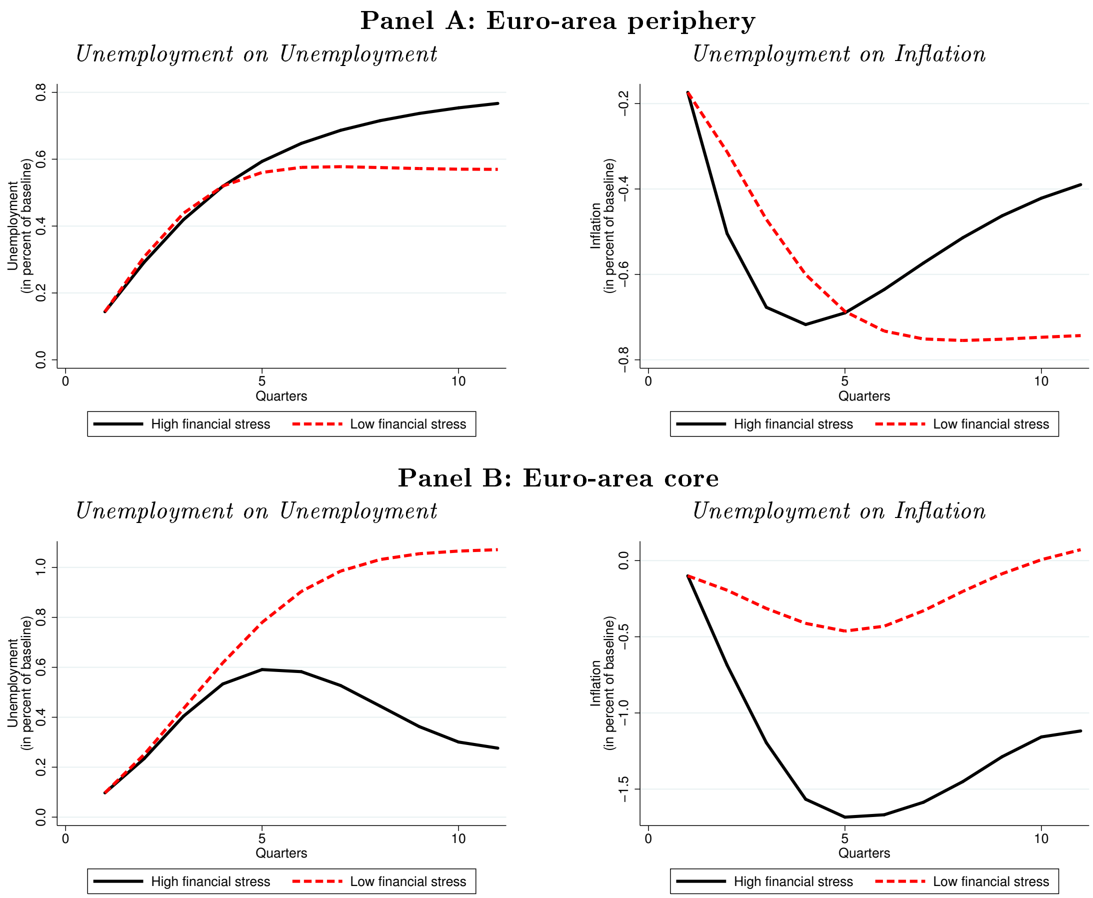


Note: The blue line represents normal financial conditions, the red line excessive debt.

Figure 15: Impulse-response functions in different financial regimes: Euro area

Figure 16 compares the reaction of both the unemployment and the inflation rate with respect to a positive unemployment rate shock for the two regimes of financial stress. For the two different regimes Panel A depicts results for the Euro-area periphery country group whereas Panel B presents the results for the Euro-area core country group. The blue lines depict the reaction of the economy in periods of low financial stress, the green the reaction when financial stress is high.

As can be seen from the different charts, the reaction of unemployment to a adverse shock on the labor market is very similar in both regimes with unemployment increasing over the five years following the shock. Small differences exist between the two regimes in the Euro-area core country group where, notably, the unemployment rate rises further when financial stress is low. More significant are, however, the differences in inflation dynamics between the two regimes. In both country groups, inflation declines much more strongly in periods of high financial stress. Note that the disinflationary pressure is even stronger in the high financial stress regime among Euro-area core countries, highlighting one of the challenges to Euro-area periphery countries have been facing during the crisis: Despite record-high unemployment rates, inflation rates fell only very gradually such as to restore competitiveness in these countries. Our estimates reflect this strong persistence of inflation dynamics in these countries, as discussed in sects. 2 and 3.

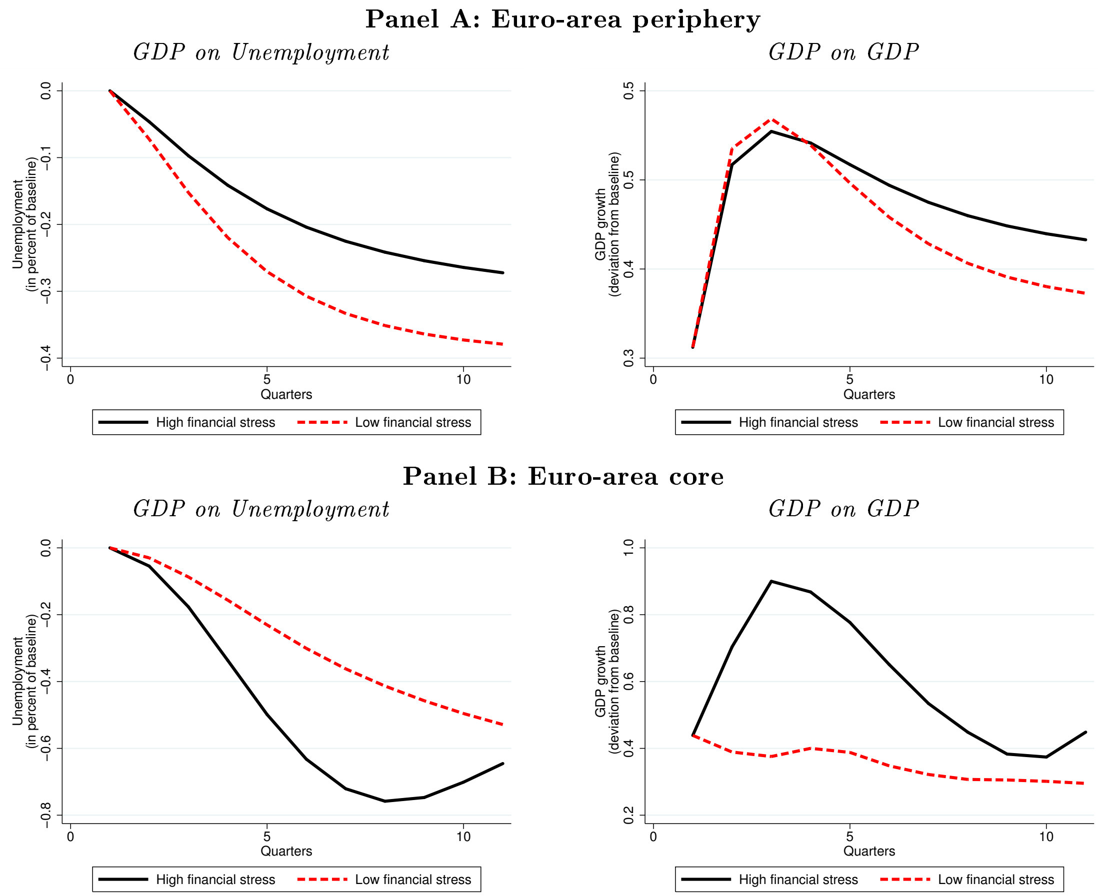


Note: Low financial stress regimes are represented in blue, high financial stress regimes in green.

Figure 16: Regime-dependent Phillips curve, ZEW-FCI as exogenous transition variable: Euro-area core vs. periphery

5.3 Regime-dependent Okun's law

Finally, we want to present results that also confirm some nonlinearities with respect to output growth and unemployment reduction. This relationship has been called Okun's law, that output growth drives employment and thus reduces unemployment. We want to explore here, again using our VSTAR method, with the ZEW FCI as transition variable, whether the unemployment reduction is also regime dependent, whereby the regime is again defined by high or low financial stress.



Note: Low financial stress regimes are represented in blue, high financial stress regimes in green.

Figure 17: Regime-dependent Okun's law, ZEW-FCI as exogenous transition variable: Euro area core vs. periphery

Overall, as can be seen from the two left graphs of Panel A and B of figure 17, a positive shock on output reduces the unemployment more in the periphery countries than in the core countries (see the scale), but if there is financial stress in the periphery countries, the growth has a larger success on reduction of unemployment if the growth rate is increased. Moreover, the second round effect of growth on growth is stronger in the South than in the North. This means that a higher growth rates in the Southern countries are useful targets to bring up employment in the South.

6. Data

We are estimating a 4 dimensional MRVAR with change (first difference) in GDP, inflation rate, interest rate and the ZEW FCI index as the endogenous threshold variable. The dataset runs from 1980 until 2013 on a quarterly basis.

Three different sources were utilized for constructing our dataset: we use the data provided by the GVAR project (Smith and Galesi, 2014) for change in GDP and inflation, while the GVAR data, together with the MFI interest rate statistics of the ECB[83], were used for computing the

[83]https://sdw.ecb.europa.eu/browse.do?node=2018774

interest rate. Finally, the ZEW FCI index was taken from Schleer and Semmler (2014) and is described in detail in appendix 5.

Smith and Galesi (2014) obtain their data from the International Financial Statistics (IFS) provided by the IMF. GDP is a real index with base year 2005 (concept: Gross Domestic Product, Real Index, Quarterly, 2005 = 100), while the inflation rate represents changes in consumer prices (concept: Consumer Prices, All items, Quarterly, 2005 = 100).

We also utilize the long-term interest rate of the GVAR project (concept: Interest Rates, Government Securities, Government Bonds concept) and augment it with long-term borrowing from the MFI interest rate statistics. The interest rate was computed the following way: from 1980 until 2003 we use the GVAR data which has been detrended by a linear trend. From 2003 until 2013 the GVAR interest rate is substituted for the MFI interest rate.

7. Estimation Results

The tables below show the estimation results of the MRVAR for the four countries.

$$\left(X_t^1 \right) = \begin{cases} \left(0.0101 \right) + \left(0.7468 \right) \left(X_{t-1}^1 \right) & \text{if Th} < 2.47330118349762 \\ \left(0.0025 \right) + \left(0.6542 \right) \left(X_{t-1}^1 \right) & \text{if Th} > 2.47330118349762 \end{cases} \tag{28}$$

Table 5: MRVAR for Spain

$$\left(X_t^1 \right) = \begin{cases} \left(0.0237 \right) + \left(-0.0130 \right) \left(X_{t-1}^1 \right) & \text{if Th} < -0.116980175503211 \\ \left(0.0202 \right) + \left(0.3552 \right) \left(X_{t-1}^1 \right) & \text{if Th} > -0.116980175503211 \end{cases} \tag{29}$$

Table 6: MRVAR for Italy

$$\left(X_t^1 \right) = \begin{cases} \left(0.0167 \right) + \left(-0.0151 \right) \left(X_{t-1}^1 \right) & \text{if Th} < 1.23323665364252 \\ \left(0.0193 \right) + \left(0.0198 \right) \left(X_{t-1}^1 \right) & \text{if Th} > 1.23323665364252 \end{cases} \tag{30}$$

Table 7: MRVAR for Germany

$$\left(X_t^1 \right) = \begin{cases} \left(0.0075 \right) + \left(0.2060 \right) \left(X_{t-1}^1 \right) & \text{if Th} < 1.70001261995449 \\ \left(0.0350 \right) + \left(0.1765 \right) \left(X_{t-1}^1 \right) & \text{if Th} > 1.70001261995449 \end{cases} \tag{31}$$

Table 8: MRVAR for France

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